The GraphBLAS C API Specification [†]:

Version 2.0.0

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[†]Based on *GraphBLAS Mathematics* by Jeremy Kepner

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Chapter 1

Introduction

The GraphBLAS standard defines a set of matrix and vector operations based on semiring algebraic structures. These operations can be used to express a wide range of graph algorithms. This document defines the C binding to the GraphBLAS standard. We refer to this as the *GraphBLAS C API* (Application Programming Interface).

The GraphBLAS C API is built on a collection of objects exposed to the C programmer as opaque data types. Functions that manipulate these objects are referred to as *methods*. These methods fully define the interface to GraphBLAS objects to create or destroy them, modify their contents, and copy the contents of opaque objects into non-opaque objects; the contents of which are under direct control of the programmer.

The GraphBLAS C API is designed to work with C99 (ISO/IEC 9899:199) extended with *static type-based* and *number of parameters-based* function polymorphism, and language extensions on par with the `_Generic` construct from C11 (ISO/IEC 9899:2011). Furthermore, the standard assumes programs using the GraphBLAS C API will execute on hardware that supports floating point arithmetic such as that defined by the IEEE 754 (IEEE 754-2008) standard.

The GraphBLAS C API assumes programs will run on a system that supports acquire-release memory orders. This is needed to support the memory models required for multithreaded execution as described in section 2.5.2.

Implementations of the GraphBLAS C API will target a wide range of platforms. We expect cases will arise where it will be prohibitive for a platform to support a particular type or a specific parameter for a method defined by the GraphBLAS C API. We want to encourage implementors to support the GraphBLAS C API even when such cases arise. Hence, an implementation may still call itself "conformant" as long as the following conditions hold.

- Every method and operation from chapter 4 is supported for the vast majority of cases.
- Any cases not supported must be documented as an implementation-defined feature of the GraphBLAS implementation. Unsupported cases must be caught as an API error (section 2.6) with the parameter GrB_NOT_IMPLEMENTED returned by the associated method call.
- It is permissible to omit the corresponding nonpolymorphic methods from chapter 5 when it

is not possible to express the signature of that method.

The number of allowed omitted cases is vague by design. We cannot anticipate the features of target platforms, on the market today or in the future, that might cause problems for the GraphBLAS specification. It is our expectation, however, that such omitted cases would be a minuscule fraction of the total combination of methods, types, and parameters defined by the GraphBLAS C API specification.

The remainder of this document is organized as follows:

- Chapter 2: Basic Concepts
- Chapter 3: Objects
- Chapter 4: Methods
- Chapter 5: Nonpolymorphic interface
- Appendix A: Revision history
- Appendix B: Non-opaque data format definitions
- Appendix C: Examples

Chapter 2

Basic concepts

The GraphBLAS C API is used to construct graph algorithms expressed "in the language of linear algebra." Graphs are expressed as matrices, and the operations over these matrices are generalized through the use of a semiring algebraic structure.

In this chapter, we will define the basic concepts used to define the GraphBLAS C API. We provide the following elements:

- Glossary of terms and notation used in this document.
- Algebraic structures and associated arithmetic foundations of the API.
- Functions that appear in the GraphBLAS algebraic structures and how they are managed.
- Domains of elements in the GraphBLAS.
- Indices, index arrays, scalar arrays, and external matrix formats used to expose the contents of GraphBLAS objects.
- The GraphBLAS opaque objects.
- The execution and error models implied by the GraphBLAS C specification.
- Enumerations used by the API and their values.

2.1 Glossary

2.1.1 GraphBLAS API basic definitions

- *application*: A program that calls methods from the GraphBLAS C API to solve a problem.
- *GraphBLAS C API*: The application programming interface that fully defines the types, objects, literals, and other elements of the C binding to the GraphBLAS.

- *function*: Refers to a named group of statements in the C programming language. Methods, operators, and user-defined functions are typically implemented as C functions. When referring to the code programmers write, as opposed to the role of functions as an element of the GraphBLAS, they may be referred to as such.

- *method*: A function defined in the GraphBLAS C API that manipulates GraphBLAS objects or other opaque features of the implementation of the GraphBLAS API.

- *operator*: A function that performs an operation on the elements stored in GraphBLAS matrices and vectors.

- *GraphBLAS operation*: A mathematical operation defined in the GraphBLAS mathematical specification. These operations (not to be confused with *operators*) typically act on matrices and vectors with elements defined in terms of an algebraic semiring.

2.1.2 GraphBLAS objects and their structure

- *non-opaque datatype*: Any datatype that exposes its internal structure and can be manipulated directly by the user.

- *opaque datatype*: Any datatype that hides its internal structure and can be manipulated only through an API.

- *GraphBLAS object*: An instance of an *opaque datatype* defined by the *GraphBLAS C API* that is manipulated only through the GraphBLAS API. There are four kinds of GraphBLAS opaque objects: *domains* (i.e., types), *algebraic objects* (operators, monoids and semirings), *collections* (scalars, vectors, matrices and masks), and descriptors.

- *handle*: A variable that holds a reference to an instance of one of the GraphBLAS opaque objects. The value of this variable holds a reference to a GraphBLAS object but not the contents of the object itself. Hence, assigning a value to another variable copies the reference to the GraphBLAS object of one handle but not the contents of the object.

- *domain*: The set of valid values for the elements stored in a GraphBLAS *collection* or operated on by a GraphBLAS *operator*. Note that some GraphBLAS objects involve functions that map values from one or more input domains onto values in an output domain. These GraphBLAS objects would have multiple domains.

- *collection*: An opaque GraphBLAS object that holds a number of elements from a specified *domain*. Because these objects are based on an opaque datatype, an implementation of the GraphBLAS C API has the flexibility to optimize the data structures for a particular platform. GraphBLAS objects are often implemented as sparse data structures, meaning only the subset of the elements that have values are stored.

- *implied zero*: Any element that has a valid index (or indices) in a GraphBLAS vector or matrix but is not explicitly identified in the list of elements of that vector or matrix. From a mathematical perspective, an *implied zero* is treated as having the value of the zero element of the relevant monoid or semiring. However, GraphBLAS operations are purposefully defined using set notation in such a way that it makes it unnecessary to reason about implied zeros. Therefore, this concept is not used in the definition of GraphBLAS methods and operators.

- *mask*: An internal GraphBLAS object used to control how values are stored in a method's output object. The mask exists only inside a method; hence, it is called an *internal opaque object*. A mask is formed from the elements of a collection object (vector or matrix) input as a mask parameter to a method. GraphBLAS allows two types of masks:

  1. In the default case, an element of the mask exists for each element that exists in the input collection object when the value of that element, when cast to a Boolean type, evaluates to `true`.
  2. In the *structure only* case, masks have structure but no values. The input collection describes a structure whereby an element of the mask exists for each element stored in the input collection regardless of its value.

- *complement*: The *complement* of a GraphBLAS mask, M, is another mask, M', where the elements of M' are those elements from M that *do not* exist.

2.1.3 Algebraic structures used in the GraphBLAS

- *associative operator*: In an expression where a binary operator is used two or more times consecutively, that operator is *associative* if the result does not change regardless of the way operations are grouped (without changing their order). In other words, in a sequence of binary operations using the same associative operator, the legal placement of parenthesis does not change the value resulting from the sequence operations. Operators that are associative over infinitely precise numbers (e.g., real numbers) are not strictly associative when applied to numbers with finite precision (e.g., floating point numbers). Such non-associativity results, for example, from roundoff errors or from the fact some numbers can not be represented exactly as floating point numbers. In the GraphBLAS specification, as is common practice in computing, we refer to operators as *associative* when their mathematical definition over infinitely precise numbers is associative even when they are only approximately associative when applied to finite precision numbers.

  No GraphBLAS method will imply a predefined grouping over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.

- *commutative operator*: In an expression where a binary operator is used (usually two or more times consecutively), that operator is *commutative* if the result does not change regardless of the order the inputs are operated on.

  No GraphBLAS method will imply a predefined ordering over any commutative operators. Implementations of the GraphBLAS are encouraged to exploit commutativity to optimize performance of any GraphBLAS method with this requirement. This holds even if the definition of the GraphBLAS method implies a fixed order for the commutative operations.

- *GraphBLAS operators*: Binary or unary operators that act on elements of GraphBLAS objects. *GraphBLAS operators* are used to express algebraic structures used in the GraphBLAS such as monoids and semirings. They are also used as arguments to several GraphBLAS methods. There are two types of *GraphBLAS operators*: (1) predefined operators found in Table 3.5 and (2) user-defined operators created using `GrB_UnaryOp_new()` or `GrB_BinaryOp_new()` (see Section 4.2.1).

- *monoid*: An algebraic structure consisting of one domain, an associative binary operator, and the identity of that operator. There are two types of GraphBLAS monoids: (1) predefined monoids found in Table 3.7 and (2) user-defined monoids created using `GrB_Monoid_new()` (see Section 4.2.1).

- *semiring*: An algebraic structure consisting of a set of allowed values (the *domain*), a commutative and associative binary operator called addition, a binary operator called multiplication (where multiplication distributes over addition), and identities over addition (*0*) and multiplication (*1*). The additive identity is an annihilator over multiplication.

- *GraphBLAS semiring*: is allowed to diverge from the mathematically rigorous definition of a *semiring* since certain combinations of domains, operators, and identity elements are useful in graph algorithms even when they do not strictly match the mathematical definition of a semiring. There are two types of *GraphBLAS semirings*: (1) predefined semirings found in Tables 3.8 and 3.9, and (2) user-defined semirings created using `GrB_Semiring_new()` (see Section 4.2.1).

- *index unary operator*: A variation of the unary operator that operates on elements of GraphBLAS vectors and matrices along with the index values representing their location in the objects. There are predefined index unary operators found in Table 3.6), and user-defined operators created using `GrB_IndexUnaryOp_new` (see Section 4.2.1).

2.1.4 The execution of an application using the GraphBLAS C API

- *program order*: The order of the GraphBLAS method calls in a thread, as defined by the text of the program.

- *host programming environment*: The GraphBLAS specification defines an API. The functions from the API appear in a program. This program is written using a programming language and execution environment defined outside of the GraphBLAS. We refer to this programming environment as the "host programming environment".

- *execution time*: time expended while executing instructions defined by a program. This term is specifically used in this specification in the context of computations carried out on behalf of a call to a GraphBLAS method.

- *sequence*: A GraphBLAS application uniquely defines a directed acyclic graph (DAG) of GraphBLAS method calls based on their program order. At any point in a program, the state of any GraphBLAS object is defined by a subgraph of that DAG. An ordered collection of GraphBLAS method calls in program order that defines that subgraph for a particular object is the *sequence* for that object.

- *complete*: A GraphBLAS object is complete when it can be used in a happens-before relationship with a method call that reads the variable on another thread. This concept is used when reasoning about memory orders in multithreaded programs. A GraphBLAS object defined on one thread that is complete can be safely used as an IN or INOUT argument in a method-call on a second thread assuming the method calls are correctly synchronized so the definition on the first thread *happens-before* it is used on the second thread. In blocking-mode, an object is complete after a GraphBLAS method call that writes to that object returns. In nonblocking-mode, an object is complete after a call to the GrB_wait() method with the GrB_COMPLETE parameter.

- *materialize*: A GraphBLAS object is materialized when it is (1) complete, (2) the computations defined by the sequence that define the object have finished (either fully or stopped at an error) and will not consume any additional computational resources, and (3) any errors associated with that sequence are available to be read according to the GraphBLAS error model. A GraphBLAS object that is never loaded into a non-opaque data structure may potentially never be materialized. This might happen, for example, if the operations associated with the object are fused or otherwise changed by the runtime system that supports the implementation of the GraphBLAS C API. An object can be materialized by a call to the materialize mode of the GrB_wait() method.

- *context*: An instance of the GraphBLAS C API implementation as seen by an application. An application can have only one context between the start and end of the application. A context begins with the first thread that calls GrB_init() and ends with the first thread to call GrB_finalize(). It is an error for GrB_init() or GrB_finalize() to be called more than one time within an application. The context is used to constrain the behavior of an instance of the GraphBLAS C API implementation and support various execution strategies. Currently, the only supported constraints on a context pertain to the mode of program execution.

- *program execution mode*: Defines how a GraphBLAS sequence executes, and is associated with the *context* of a GraphBLAS C API implementation. It is set by an application with its call to GrB_init() to one of two possible states. In *blocking mode*, GraphBLAS methods return after the computations complete and any output objects have been materialized. In *nonblocking mode*, a method may return once the arguments are tested as consistent with the method (i.e., there are no API errors), and potentially before any computation has taken place.

2.1.5 GraphBLAS methods: behaviors and error conditions

- *implementation-defined behavior*: Behavior that must be documented by the implementation and is allowed to vary among different compliant implementations.

- *undefined behavior*: Behavior that is not specified by the GraphBLAS C API. A conforming implementation is free to choose results delivered from a method whose behavior is undefined.

- *thread-safe*: Consider a function called from multiple threads with arguments that do not overlap in memory (i.e. the argument lists do not share memory). If the function is *thread-safe* then it will behave the same when executed concurrently by multiple threads or sequentially on a single thread.

- *dimension compatible*: GraphBLAS objects (matrices and vectors) that are passed as parameters to a GraphBLAS method are dimension (or shape) compatible if they have the correct number of dimensions and sizes for each dimension to satisfy the rules of the mathematical definition of the operation associated with the method. If any *dimension compatibility* rule above is violated, execution of the GraphBLAS method ends and the GrB_DIMENSION_MISMATCH error is returned.

- *domain compatible*: Two domains for which values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other, and a domain from a user-defined type is only compatible with itself. If any *domain compatibility* rule above is violated, execution of the GraphBLAS method ends and the GrB_DOMAIN_MISMATCH error is returned.

472 ## 2.2 Notation

Notation	Description
$D_{out}, D_{in}, D_{in_1}, D_{in_2}$	Refers to output and input domains of various GraphBLAS operators.
$\mathbf{D}_{out}(*), \mathbf{D}_{in}(*), \mathbf{D}_{in_1}(*), \mathbf{D}_{in_2}(*)$	Evaluates to output and input domains of GraphBLAS operators (usually a unary or binary operator, or semiring).
$\mathbf{D}(*)$	Evaluates to the (only) domain of a GraphBLAS object (usually a monoid, vector, or matrix).
f	An arbitrary unary function, usually a component of a unary operator.
$\mathbf{f}(F_u)$	Evaluates to the unary function contained in the unary operator given as the argument.
$\odot$	An arbitrary binary function, usually a component of a binary operator.
$\bigodot(*)$	Evaluates to the binary function contained in the binary operator or monoid given as the argument.
$\otimes$	Multiplicative binary operator of a semiring.
$\oplus$	Additive binary operator of a semiring.
$\bigotimes(S)$	Evaluates to the multiplicative binary operator of the semiring given as the argument.
$\bigoplus(S)$	Evaluates to the additive binary operator of the semiring given as the argument.
$\mathbf{0}(*)$	The identity of a monoid, or the additive identity of a GraphBLAS semiring.
$\mathbf{L}(*)$	The contents (all stored values) of the vector or matrix GraphBLAS objects. For a vector, it is the set of (index, value) pairs, and for a matrix it is the set of (row, col, value) triples.
473 $\mathbf{v}(i)$ or v_i	The i^{th} element of the vector $\mathbf{v}$.
$\mathbf{size}(\mathbf{v})$	The size of the vector $\mathbf{v}$.
$\mathbf{ind}(\mathbf{v})$	The set of indices corresponding to the stored values of the vector $\mathbf{v}$.
$\mathbf{nrows}(\mathbf{A})$	The number of rows in the $\mathbf{A}$.
$\mathbf{ncols}(\mathbf{A})$	The number of columns in the $\mathbf{A}$.
$\mathbf{indrow}(\mathbf{A})$	The set of row indices corresponding to rows in $\mathbf{A}$ that have stored values.
$\mathbf{indcol}(\mathbf{A})$	The set of column indices corresponding to columns in $\mathbf{A}$ that have stored values.
$\mathbf{ind}(\mathbf{A})$	The set of (i, j) indices corresponding to the stored values of the matrix.
$\mathbf{A}(i, j)$ or A_{ij}	The element of $\mathbf{A}$ with row index i and column index j.
$\mathbf{A}(:, j)$	The j^{th} column of matrix $\mathbf{A}$.
$\mathbf{A}(i, :)$	The i^{th} row of matrix $\mathbf{A}$.
$\mathbf{A}^T$	The transpose of matrix $\mathbf{A}$.
$\neg\mathbf{M}$	The complement of $\mathbf{M}$.
$\mathrm{s}(\mathbf{M})$	The structure of $\mathbf{M}$.
$\widetilde{\mathbf{t}}$	A temporary object created by the GraphBLAS implementation.
$< type >$	A method argument type that is `void *` or one of the types from Table 3.2.
GrB_ALL	A method argument literal to indicate that all indices of an input array should be used.
GrB_Type	A method argument type that is either a user defined type or one of the types from Table 3.2.
GrB_Object	A method argument type referencing any of the GraphBLAS object types.
GrB_NULL	The GraphBLAS NULL.

2.3 Mathematical foundations

Graphs can be represented in terms of matrices. The values stored in these matrices correspond to attributes (often weights) of edges in the graph.[1] Likewise, information about vertices in a graph are stored in vectors. The set of valid values that can be stored in either matrices or vectors is referred to as their domain. Matrices are usually sparse because the lack of an edge between two vertices means that nothing is stored at the corresponding location in the matrix. Vectors may be sparse or dense, or they may start out sparse and become dense as algorithms traverse the graphs.

Operations defined by the GraphBLAS C API specification operate on these matrices and vectors to carry out graph algorithms. These GraphBLAS operations are defined in terms of GraphBLAS semiring algebraic structures. Modifying the underlying semiring changes the result of an operation to support a wide range of graph algorithms. Inside a given algorithm, it is often beneficial to change the GraphBLAS semiring that applies to an operation on a matrix. This has two implications for the C binding of the GraphBLAS API.

First, it means that we define a separate object for the semiring to pass into methods. Since in many cases the full semiring is not required, we also support passing monoids or even binary operators, which means the semiring is implied rather than explicitly stated.

Second, the ability to change semirings impacts the meaning of the *implied zero* in a sparse representation of a matrix or vector. This element in real arithmetic is zero, which is the identity of the *addition* operator and the annihilator of the *multiplication* operator. As the semiring changes, this implied zero changes to the identity of the *addition* operator and the annihilator (if present) of the *multiplication* operator for the new semiring. Nothing changes regarding what is stored in the sparse matrix or vector, but the implied zeros within them change with respect to a particular operation. In all cases, the nature of the implied zero does not matter since the GraphBLAS C API requires that implementations treat them as nonexistent elements of the matrix or vector.

As with matrices and vectors, GraphBLAS semirings have domains associated with their inputs and outputs. The semirings in the GraphBLAS C API are defined with two domains associated with the input operands and one domain associated with output. When used in the GraphBLAS C API these domains may not match the domains of the matrices and vectors supplied in the operations. In this case, only valid *domain compatible* casting is supported by the API.

The mathematical formalism for graph operations in the language of linear algebra often assumes that we can operate in the field of real numbers. However, the GraphBLAS C binding is designed for implementation on computers, which by necessity have a finite number of bits to represent numbers. Therefore, we require a conforming implementation to use floating point numbers such as those defined by the IEEE-754 standard (both single- and double-precision) wherever real numbers need to be represented. The practical implications of these finite precision numbers is that the result of a sequence of computations may vary from one execution to the next as the grouping of operands (because of associativity) within the operations changes. While techniques are known to reduce these effects, we do not require or even expect an implementation to use them as they may add

[1] More information on the mathematical foundations can be found in the following paper: J. Kepner, P. Aaltonen, D. Bader, A. Buluç, F. Franchetti, J. Gilbert, D. Hutchison, M. Kumar, A. Lumsdaine, H. Meyerhenke, S. McMillan, J. Moreira, J. Owens, C. Yang, M. Zalewski, and T. Mattson. 2016, September. Mathematical foundations of the GraphBLAS. In *2016 IEEE High Performance Extreme Computing Conference (HPEC)* (pp. 1-9). IEEE.

Table 2.1: Types of GraphBLAS opaque objects.

GrB_Object types	Description
GrB_Type	Scalar type.
GrB_UnaryOp	Unary operator.
GrB_IndexUnaryOp	Unary operator, that operates on a single value and its location index values.
GrB_BinaryOp	Binary operator.
GrB_Monoid	Monoid algebraic structure.
GrB_Semiring	A GraphBLAS semiring algebraic structure.
GrB_Scalar	One element; could be empty.
GrB_Vector	One-dimensional collection of elements; can be sparse.
GrB_Matrix	Two-dimensional collection of elements; typically sparse.
GrB_Descriptor	Descriptor object, used to modify behavior of methods (specifically GraphBLAS operations).

considerable overhead. In most cases, these roundoff errors are not significant. When they are significant, the problem itself is ill-conditioned and needs to be reformulated.

2.4 GraphBLAS opaque objects

Objects defined in the GraphBLAS standard include types (the domains of elements), collections of elements (matrices, vectors, and scalars), operators on those elements (unary, index unary, and binary operators), algebraic structures (semirings and monoids), and descriptors. GraphBLAS objects are defined as opaque types; that is, they are managed, manipulated, and accessed solely through the GraphBLAS application programming interface. This gives an implementation of the GraphBLAS C specification flexibility to optimize objects for different scenarios or to meet the needs of different hardware platforms.

A GraphBLAS opaque object is accessed through its *handle*. A handle is a variable that references an instance of one of the types from Table 2.1. An implementation of the GraphBLAS specification has a great deal of flexibility in how these handles are implemented. All that is required is that the handle corresponds to a type defined in the C language that supports assignment and comparison for equality. The GraphBLAS specification defines a literal GrB_INVALID_HANDLE that is valid for each type. Using the logical equality operator from C, it must be possible to compare a handle to GrB_INVALID_HANDLE to verify that a handle is valid.

Every GraphBLAS object has a *lifetime*, which consists of the sequence of instructions executed in program order between the *creation* and the *destruction* of the object. The GraphBLAS C API predefines a number of these objects which are created when the GraphBLAS context is initialized by a call to GrB_init and are destroyed when the GraphBLAS context is terminated by a call to GrB_finalize.

An application using the GraphBLAS API can create additional objects by declaring variables of the appropriate type from Table 2.1 for the objects it will use. Before use, the object must be initialized with a call call to one of the object's respective *constructor* methods. Each kind of object has at least one explicit constructor method of the form GrB_*_new where '*' is replaced with the type of object (e.g., GrB_Semiring_new). Note that some objects, especially collections, have additional constructor methods such as duplication, import, or deserialization. Objects explicitly created by a call to a constructor should be destroyed by a call to GrB_free. The behavior of a program that calls GrB_free on a pre-defined object is undefined.

These constructor and destructor methods are the only methods that change the value of a handle. Hence, objects changed by these methods are passed into the method as pointers. In all other cases, handles are not changed by the method and are passed by value. For example, even when multiplying matrices, while the contents of the output product matrix changes, the handle for that matrix is unchanged.

Several GraphBLAS constructor methods take other objects as input arguments and use these objects to create a new object. For all these methods, the lifetime of the created object must end strictly before the lifetime of any dependent input objects. For example, a vector constructor GrB_Vector_new takes a GrB_Type object as input. That type object must not be destroyed until after the created vector is destroyed. Similarly, a GrB_Semiring_new method takes a monoid and a binary operator as inputs. Neither of these can be destroyed until after the created semiring is destroyed.

Note that some constructor methods like GrB_Vector_dup and GrB_Matrix_dup behave differently. In these cases, the input vector or matrix can be destroyed as soon as the call returns. However, the original type object used to create the input vector or matrix cannot be destroyed until after the vector or matrix created by GrB_Vector_dup or GrB_Matrix_dup is destroyed. This behavior must hold for any chain of duplicating constructors.

Programmers using GraphBLAS handles must be careful to distinguish between a handle and the object manipulated through a handle. For example, a program may declare two GraphBLAS objects of the same type, initialize one, and then assign it to the other variable. That assignment, however, only assigns the handle to the variable. It does not create a copy of that variable (to do that, one would need to use the appropriate duplication method). If later the object is freed by calling GrB_free with the first variable, the object is destroyed and the second variable is left referencing an object that no longer exists (a so-called "dangling handle").

In addition to opaque objects manipulated through handles, the GraphBLAS C API defines an additional opaque object as an internal object; that is, the object is never exposed as a variable within an application. This opaque object is the mask used to control which computed values can be stored in the output operand of a *GraphBLAS operation*. Masks are described in Section 3.5.4.

2.5 Execution model

A program using the GraphBLAS C API is called a GraphBLAS application. The application constructs GraphBLAS objects, manipulates them to implement a graph algorithm, and then extracts values from the GraphBLAS objects to produce the results for that algorithm. Functions defined within the GraphBLAS C API that manipulate GraphBLAS objects are called *methods*. If the method corresponds to one of the operations defined in the GraphBLAS mathematical specification, we refer to the method as an *operation*.

The GraphBLAS application specifies an ordered collection of GraphBLAS method calls defined by the order they appear in the text of the program (the *program order*). These define a directed acyclic graph (DAG) where nodes are GraphBLAS method calls and edges are dependencies between method calls.

Each method call in the DAG uniquely and unambiguously defines the output GraphBLAS objects as long as there are no execution errors that put objects in an invalid state (see Section 2.6). An ordered collection of method calls, a subgraph of the overall DAG for an application, defines the state of a GraphBLAS object at any point in a program. This ordered collection is the *sequence* for that object.

Since the GraphBLAS execution is defined in terms of a DAG and the GraphBLAS objects are opaque, the semantics of the GraphBLAS specification affords an implementation considerable flexibility to optimize performance. A GraphBLAS implementation can defer execution of nodes in the DAG, fuse nodes, or even replace whole subgraphs within the DAG to optimize performance. We discuss this topic further in section 2.5.1 when we describe *blocking* and *non-blocking* execution modes.

A correct GraphBLAS application must be *race-free*. This means that the DAG produced by an application and the results produced by execution of that DAG must be the same regardless of how the threads are scheduled for execution. It is the application programmer's responsibility to control memory orders and establish the required synchronized-with relationships to assure race-free execution of a multi-threaded GraphBLAS application. Writing race-free GraphBLAS applications is discussed further in Section 2.5.2.

2.5.1 Execution modes

The execution of the DAG defined by a GraphBLAS application depends on the *execution mode* of the GraphBLAS program. There are two modes: *blocking* and *nonblocking*.

- *blocking*: In blocking mode, each method finishes the GraphBLAS operation defined by the method and all output GraphBLAS objects are *materialized* before proceeding to the next statement. Even mechanisms that break the opaqueness of the GraphBLAS objects (e.g., performance monitors, debuggers, memory dumps) will observe that the operation has finished.

- *nonblocking*: In nonblocking mode, each method may return once the input arguments have been inspected and verified to define a well formed GraphBLAS operation. (That is, there are no API errors; see Section 2.6.) The GraphBLAS method may not have finished, but the output object is ready to be used by the next GraphBLAS method call. If needed, a call to GrB_wait with GrB_COMPLETE or GrB_MATERIALIZE can be used to force the sequence for a GraphBLAS object (obj) to finish its execution.

The *execution mode* is defined in the GraphBLAS C API when the context of the library invocation is defined. This occurs once before any GraphBLAS methods are called with a call to the

GrB_init() function. This function takes a single argument of type GrB_Mode with values shown in Table 3.1(a).

An application executing in nonblocking mode is not required to return immediately after input arguments have been verified. A conforming implementation of the GraphBLAS C API running in nonblocking mode may choose to execute *as if* in blocking mode. A sequence of operations in nonblocking mode where every GraphBLAS operation with output object obj is followed by a GrB_wait(obj, GrB_MATERIALIZE) call is equivalent to the same sequence in blocking mode with GrB_wait(obj, GrB_MATERIALIZE) calls removed.

Nonblocking mode allows for any execution strategy that satisfies the mathematical definition of the sequence. The methods can be placed into a queue and deferred. They can be chained together and fused (e.g., replacing a chained pair of matrix products with a matrix triple product). Lazy evaluation, greedy evaluation, and asynchronous execution are all valid as long as the final result agrees with the mathematical definition provided by the sequence of GraphBLAS method calls appearing in program order.

Blocking mode forces an implementation to carry out precisely the GraphBLAS operations defined by the methods and to complete each and every method call individually. It is valuable for debugging or in cases where an external tool such as a debugger needs to evaluate the state of memory during a sequence of operations.

In a sequence of operations free of execution errors, and with input objects that are well-conditioned, the results from blocking and nonblocking modes should be identical outside of effects due to roundoff errors associated with floating point arithmetic. Due to the great flexibility afforded to an implementation when using nonblocking mode, we expect execution of a sequence in nonblocking mode to potentially complete execution in less time.

It is important to note that, processing of nonopaque objects is never deferred in GraphBLAS. That is, methods that consume nonopaque objects (e.g., GrB_Matrix_build(), Section 4.2.4.9) and methods that produce nonopaque objects (e.g., GrB_Matrix_extractTuples(), Section 4.2.4.13) always finish consuming or producing those nonopaque objects before returning regardless of the execution mode.

Finally, after all GraphBLAS method calls have been made, the context is terminated with a call to GrB_finalize(). In the current version of the GraphBLAS C API, the context can be set only once in the execution of a program. That is, after GrB_finalize() is called, a subsequent call to GrB_init() is not allowed.

2.5.2 Multi-threaded execution

The GraphBLAS C API is designed to work with applications that utilize multiple threads executing within a shared address space. This specification does not define how threads are created, managed and synchronized. We expect the host programming environment to provide those services.

A conformant implementation of the GraphBLAS must be *thread safe*. A GraphBLAS library is thread safe when independent method calls (i.e., GraphBLAS objects are not shared between method calls) from multiple threads in a race-free program return the same results as would follow from their sequential execution in some interleaved order. This is a common requirement in software libraries.

Thread safety applies to the behavior of multiple independent threads. In the more general case for multithreading, threads are not independent; they share variables and mix read and write operations to those variables across threads. A memory consistency model defines which values can be returned when reading an object shared between two or more threads. The GraphBLAS specification does not define its own memory consistency model. Instead the specification defines what must be done by a programmer calling GraphBLAS methods and by the implementor of a GraphBLAS library so an implementation of the GraphBLAS specification can work correctly with the memory consistency model for the host environment.

A memory consistency model is defined in terms of happens-before relations between methods in different threads. The defining case is a method that writes to an object on one thread that is read (i.e., used as an IN or INOUT argument) in a GraphBLAS method on a different thread. The following steps must occur between the different threads.

- A sequence of GraphBLAS methods results in the definition of the GraphBLAS object.
- The GraphBLAS object is put into a state of completion by a call to GrB_wait() with the GrB_COMPLETE parameter (see Table 3.1(b)). A GraphBLAS object is said to be *complete* when it can be safely used as an IN or INOUT argument in a GraphBLAS method call from a different thread.
- Completion happens before a synchronized-with relation that executes with *at least* a release memory order.
- A synchronized-with relation on the other thread executes with *at least* an acquire memory order.
- This synchronized-with relation happens-before the GraphBLAS method that reads the graphBLAS object.

We use the phrase *at least* when talking about the memory orders to indicate that a stronger memory order such as *sequential consistency* can be used in place of the acquire-release order.

A program that violates these rules contains a data race. That is, its reads and writes are unordered across threads making the final value of a variable undefined. A program that contains a data race is invalid and the results of that program are undefined. We note that multi-threaded execution is compatible with both blocking and non-blocking modes of execution.

Completion is the central concept that allows GraphBLAS objects to be used in happens-before relations between threads. In earlier versions of GraphBLAS (1.X) completion was implied by any operation that produced non-opaque values from a GraphBLAS object. These operations are summarized in Table 2.2). In GraphBLAS 2.0, these methods no longer imply completion. This change was made since there are cases where the non-opaque value is needed but the object from which it is computed is not. We want implementations of the GraphBLAS to be able to exploit this case and not form the opaque object when that object is not needed.

Table 2.2: Methods that extract values from a GraphBLAS object that forcing completion of the operations contributing to that particular object in GraphBLAS 1.X. In GraphBLAS 2.0, these methods *do not* force completion.

Method	Section
GrB_Vector_nvals	4.2.3.6
GrB_Vector_extractElement	4.2.3.10
GrB_Vector_extractTuples	4.2.3.11
GrB_Matrix_nvals	4.2.4.8
GrB_Matrix_extractElement	4.2.4.12
GrB_Matrix_extractTuples	4.2.4.13
GrB_reduce (vector-scalar value variant)	4.3.10.2
GrB_reduce (matrix-scalar value variant)	4.3.10.3

2.6 Error model

All GraphBLAS methods return a value of type GrB_Info (an enum) to provide information available to the system at the time the method returns. The returned value will be one of the defined values shown in Table 3.13. The return values fall into three groups: informational, API errors, and execution errors. While API and execution errors take on negative values, informational return values listed in Table 3.13(a) are non-negative and include GrB_SUCCESS (a value of 0) and GrB_NO_VALUE.

An API error (listed in Table 3.13(b)) means that a GraphBLAS method was called with parameters that violate the rules for that method. These errors are restricted to those that can be determined by inspecting the dimensions and domains of GraphBLAS objects, GraphBLAS operators, or the values of scalar parameters fixed at the time a method is called. API errors are deterministic and consistent across platforms and implementations. API errors are never deferred, even in nonblocking mode. That is, if a method is called in a manner that would generate an API error, it always returns with the appropriate API error value. If a GraphBLAS method returns with an API error, it is guaranteed that none of the arguments to the method (or any other program data) have been modified. The informational return value, GrB_NO_VALUE, is also deterministic and never deferred in nonblocking mode.

Execution errors (listed in Table 3.13(c)) indicate that something went wrong during the execution of a legal GraphBLAS method invocation. Their occurrence may depend on specifics of the execution environment and data values being manipulated. This does not mean that execution errors are the fault of the GraphBLAS implementation. For example, a memory leak could arise from an error in an application's source code (a "program error"), but it may manifest itself in different points of a program's execution (or not at all) depending on the platform, problem size, or what else is running at that time. Index out-of-bounds errors, for example, always indicate a program error.

If a GraphBLAS method returns with any execution error other than GrB_PANIC, it is guaranteed that the state of any argument used as input-only is unmodified. Output arguments may be left in an invalid state, and their use downstream in the program flow may cause additional errors. If a

GraphBLAS method returns with a GrB_PANIC execution error, no guarantees can be made about the state of any program data.

In nonblocking mode, execution errors can be deferred. A return value of GrB_SUCCESS only guarantees that there are no API errors in the method invocation. If an execution error value is returned by a method with output object obj in nonblocking mode, it indicates that an error was found during execution of any of the pending operations on obj, up to and including the GrB_wait() method (Section 4.2.7) call that completes those pending operations. When possible, that return value will provide information concerning the cause of the error.

As discussed in Section 4.2.7, a GrB_wait(obj) on a specific GraphBLAS object obj completes all pending operations on that object. No additional errors on the methods that precede the call to GrB_wait and have obj as an OUT or INOUT argument can be reported. From a GraphBLAS perspective, those methods are *complete*. Details on the guaranteed state of objects after a call to GrB_wait can be found in Section 4.2.7.

After a call to any GraphBLAS method that modifies an opaque object, the program can retrieve additional error information (beyond the error code returned by the method) though a call to the function GrB_error(), passing the method's output object as described in Section 4.2.8. The function returns a pointer to a NULL-terminated string, and the contents of that string are implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error string. GrB_error() is a thread-safe function, in the sense that multiple threads can call it simultaneously and each will get its own error string back, referring to the object passed as an input argument.

Chapter 3

Objects

In this chapter, all of the enumerations, literals, data types, and predefined opaque objects defined in the GraphBLAS API are presented. Enumeration literals in GraphBLAS are assigned specific values to ensure compatibility between different runtime library implementations. The chapter starts by defining the enumerations that are used by the init() and wait() methods. Then a number of transparent (i.e., non-opaque) types that are used for interfacing with external data are defined. Sections that follow describe the various types of opaque objects in GraphBLAS: types (or *domains*), algebraic objects, collections and descriptors. Each of these sections also lists the predefined instances of each opaque type that are required by the API. This chapter concludes with a section on the definition for GrB_Info enumeration that is used as the return type of all methods.

3.1 Enumerations for init() and wait()

Table 3.1 lists the enumerations and the corresponding values used in the GrB_init() method to set the execution mode and in the GrB_wait() method for completing or materializing opaque objects.

3.2 Indices, index arrays, and scalar arrays

In order to interface with third-party software (i.e., software other than an implementation of the GraphBLAS), operations such as GrB_Matrix_build (Section 4.2.4.9) and GrB_Matrix_extractTuples (Section 4.2.4.13) must specify how the data should be laid out in non-opaque data structures. To this end we explicitly define the types for indices and the arrays used by these operations.

For indices a typedef is used to give a GraphBLAS name to a concrete type. We define it as follows:

```
typedef uint64_t GrB_Index;
```

The range of valid values for a variable of type GrB_Index is [0, GrB_INDEX_MAX] where the largest index value permissible is defined with a macro, GrB_INDEX_MAX. For example:

```
#define GrB_INDEX_MAX ((GrB_Index) 0x0fffffffffffffff);
```

An implementation is required to define and document this value.

An index array is a pointer to a set of GrB_Index values that are stored in a contiguous block of memory (i.e., GrB_Index*). Likewise, a scalar array is a pointer to a contiguous block of memory storing a number of scalar values as specified by the user. Some GraphBLAS operations (e.g., GrB_assign) include an input parameter with the type of an index array. This input index array selects a subset of elements from a GraphBLAS vector or matrix object to be used in the operation. In these cases, the literal GrB_ALL can be used in place of the index array input parameter to indicate that all indices of the associated GraphBLAS vector or matrix object should be used. An implementation of the GraphBLAS C API has considerable freedom in terms of how GrB_ALL is defined. Since GrB_ALL is used as an argument for an array parameter, it must use a type consistent with a pointer. GrB_ALL must also have a non-null value to distinguish it from the erroneous case of passing a NULL pointer as an array.

3.3 Types (domains)

In GraphBLAS, domains correspond to the valid values for types from the host language (in our case, the C programming language). GraphBLAS defines a number of operators that take elements from one or more domains and produce elements of a (possibly) different domain. GraphBLAS also defines three kinds of collections: matrices, vectors and scalars. For any given collection, the elements of the collection belong to a *domain*, which is the set of valid values for the elements. For any variable or object V in GraphBLAS we denote as $\mathbf{D}(V)$ the domain of V, that is, the set of possible values that elements of V can take.

Table 3.1: Enumeration literals and corresponding values input to various GraphBLAS methods.

(a) GrB_Mode execution modes for the GrB_init method.

Symbol	Value	Description
GrB_NONBLOCKING	0	Specifies the nonblocking mode context.
GrB_BLOCKING	1	Specifies the blocking mode context.

(b) GrB_WaitMode wait modes for the GrB_wait method.

Symbol	Value	Description
GrB_COMPLETE	0	The object is in a state where it can be used in a happens-before relation so that multithreaded programs can be properly synchronized.
GrB_MATERIALIZE	1	The object is *complete*, and in addition, all computation of the object is finished and any error information is available.

Table 3.2: Predefined GrB_Type values, and the corresponding GraphBLAS domain suffixes, C type (for scalar parameters), and domains for GraphBLAS. The domain suffixes are used in place of I, F, and T in Tables 3.5, 3.6, 3.7, 3.8, and 3.9).

GrB_Type	Suffix	C type	Domain
GrB_BOOL	BOOL	`bool`	$\{\texttt{false}, \texttt{true}\}$
GrB_INT8	INT8	`int8_t`	$\mathbb{Z} \cap [-2^7, 2^7)$
GrB_UINT8	UINT8	`uint8_t`	$\mathbb{Z} \cap [0, 2^8)$
GrB_INT16	INT16	`int16_t`	$\mathbb{Z} \cap [-2^{15}, 2^{15})$
GrB_UINT16	UINT16	`uint16_t`	$\mathbb{Z} \cap [0, 2^{16})$
GrB_INT32	INT32	`int32_t`	$\mathbb{Z} \cap [-2^{31}, 2^{31})$
GrB_UINT32	UINT32	`uint32_t`	$\mathbb{Z} \cap [0, 2^{32})$
GrB_INT64	INT64	`int64_t`	$\mathbb{Z} \cap [-2^{63}, 2^{63})$
GrB_UINT64	UINT64	`uint64_t`	$\mathbb{Z} \cap [0, 2^{64})$
GrB_FP32	FP32	`float`	IEEE 754 binary32
GrB_FP64	FP64	`double`	IEEE 754 binary64

The domains for elements that can be stored in collections and operated on through GraphBLAS methods are defined by GraphBLAS objects called GrB_Type. The predefined types and corresponding domains used in the GraphBLAS C API are shown in Table 3.2. The Boolean type (`bool`) is defined in `stdbool.h`, the integral types (`int8_t`, `uint8_t`, `int16_t`, `uint16_t`, `int32_t`, `uint32_t`, `int64_t`, `uint64_t`) are defined in `stdint.h`, and the floating-point types (`float`, `double`) are native to the language and platform and in most cases defined by the IEEE-754 standard.

3.4 Algebraic objects, operators and associated functions

GraphBLAS operators operate on elements stored in GraphBLAS collections. A *binary operator* is a function that maps two input values to one output value. A *unary operator* is a function that maps one input value to one output value. Binary operators are defined over two input domains and produce an output from a (possibly different) third domain. Unary operators are specified over one input domain and produce an output from a (possibly different) second domain.

In addition to the operators that operate on stored values, GraphBLAS also supports *index unary operators* that maps a stored value and the indices of its position in the matrix or vector to an output value. That output value can be used in the index unary operator variants of apply (§ 4.3.8) to compute a new stored value, or be used in the select operation (§ 4.3.9) to determine if the stored input value should be kept or annihilated.

Some GraphBLAS operations require a monoid or semiring. A monoid contains an associative binary operator where the input and output domains are the same. The monoid also includes an identity value of the operator. The semiring consists of a binary operator – referred to as the "times" operator – with up to three different domains (two inputs and one output) and a monoid

Table 3.3: Operator input for relevant GraphBLAS operations. The semiring add and times are shown if applicable.

Operation	Operator input
mxm, mxv, vxm	semiring
eWiseAdd	binary operator monoid semiring (add)
eWiseMult	binary operator monoid semiring (times)
reduce (to vector or GrB_Scalar)	binary operator monoid
reduce (to scalar value)	monoid
apply	unary operator binary operator with scalar index unary operator
select	index unary operator
kronecker	binary operator monoid semiring
dup argument (build methods)	binary operator
accum argument (various methods)	binary operator

– referred to as the "plus" operator – that is also commutative. Furthermore, the domain of the monoid must be the same as the output domain of the "times" operator.

The GraphBLAS *algebraic objects* operators, monoids, and semirings are presented in this section. These objects can be used as input arguments to various GraphBLAS operations, as shown in Table 3.3. The specific rules for each algebraic object are explained in the respective sections of those objects. A summary of the properties and recipes for building these GraphBLAS algebraic objects is presented in Table 3.4.

A number of predefined operators are specified by the GraphBLAS C API. They are presented in tables in their respective subsections below. Each of these operators is defined to operate on specific GraphBLAS types and therefore, this type is built into the name of the object as a suffix. These suffixes and the corresponding predefined GrB_Type objects that are listed in Table 3.2.

3.4.1 Operators

A GraphBLAS *unary operator* $F_u = \langle D_{out}, D_{in}, f \rangle$ is defined by two domains, D_{out} and D_{in}, and an operation $f : D_{in} \rightarrow D_{out}$. For a given GraphBLAS unary operator $F_u = \langle D_{out}, D_{in}, f \rangle$, we define $\mathbf{D}_{out}(F_u) = D_{out}$, $\mathbf{D}_{in}(F_u) = D_{in}$, and $\mathbf{f}(F_u) = f$.

A GraphBLAS *binary operator* $F_b = \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle$ is defined by three domains, D_{out}, D_{in_1},

Table 3.4: Properties and recipes for building GraphBLAS algebraic objects: unary operator, binary operator, monoid, and semiring (composed of operations *add* and *times*).

(a) Properties of algebraic objects.

Object	Must be commutative	Must be associative	Identity must exist	Number of domains
Unary operator	n/a	n/a	n/a	2
Binary operator	no	no	no	3
Monoid	no	yes	yes	1
Reduction add	yes	yes	yes (see Note 1)	1
Semiring add	yes	yes	yes	1
Semiring times	no	no	no	3 (see Note 2)

(b) Recipes for algebraic objects.

Object	Recipe	Number of domains
Unary operator	Function pointer	2
Binary operator	Function pointer	3
Monoid	Associative binary operator with identity	1
Semiring	Commutative monoid + binary operator	3

Note 1: Some high-performance GraphBLAS implementations may require an identity to perform reductions to sparse objects like GraphBLAS vectors and scalars. According to the descriptions of the corresponding GraphBLAS operations, however, this identity is mathematically not necessary. There are API signatures to support both.
Note 2: The output domain of the semiring times must be same as the domain of the semiring's add monoid. This ensures three domains for a semiring rather than four.

D_{in_2}, and an operation $\odot : D_{in_1} \times D_{in_2} \rightarrow D_{out}$. For a given GraphBLAS binary operator $F_b = \langle D_{out}, D_{in_1}, D_{in_2}, \odot \rangle$, we define $\mathbf{D}_{out}(F_b) = D_{out}$, $\mathbf{D}_{in_1}(F_b) = D_{in_1}$, $\mathbf{D}_{in_2}(F_b) = D_{in_2}$, and $\bigodot(F_b) = \odot$. Note that $\odot$ could be used in place of either $\oplus$ or $\otimes$ in other methods and operations.

A GraphBLAS *index unary operator* $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$ is defined by three domains, D_{out}, D_{in_1}, D_{in_2}, the domain of GraphBLAS indices, and an operation $f_i : D_{in_1} \times I_{U64}^2 \times D_{in_2} \rightarrow D_{out}$ (where I_{U64} corresponds to the domain of a GrB_Index). For a given GraphBLAS index operator F_i, we define $\mathbf{D}_{out}(F_i) = D_{out}$, $\mathbf{D}_{in_1}(F_i) = D_{in_1}$, $\mathbf{D}_{in_2}(F_i) = D_{in_2}$, and $\mathbf{f}(F_i) = f_i$.

User-defined operators can be created with calls to GrB_UnaryOp_new, GrB_BinaryOp_new, and GrB_IndexUnaryOp_new, respectively. See Section 4.2.1 for information on these methods. The GraphBLAS C API predefines a number of these operators. These are listed in Tables 3.5 and 3.6. Note that most entries in these tables represent a "family" of predefined operators for a set of different types represented by the T, I, or F in their names. For example, the multiplicative inverse (GrB_MINV_F) function is only defined for floating-point types (F = FP32 or FP64). The division (GrB_DIV_T) function is defined for all types, but only if $y \neq 0$ for integral and floating point types and $y \neq$ `false` for the Boolean type.

Table 3.5: Predefined unary and binary operators for GraphBLAS in C. The T can be any suffix from Table 3.2, I can be any integer suffix from Table 3.2, and F can be any floating-point suffix from Table 3.2.

Operator type	GraphBLAS identifier	Domains	Description	
GrB_UnaryOp	GrB_IDENTITY_T	$T \to T$	$f(x) = x,$	identity
GrB_UnaryOp	GrB_ABS_T	$T \to T$	$f(x) = \|x\|,$	absolute value
GrB_UnaryOp	GrB_AINV_T	$T \to T$	$f(x) = -x,$	additive inverse
GrB_UnaryOp	GrB_MINV_F	$F \to F$	$f(x) = \frac{1}{x},$	multiplicative inverse
GrB_UnaryOp	GrB_LNOT	`bool` $\to$ `bool`	$f(x) = \neg x,$	logical inverse
GrB_UnaryOp	GrB_BNOT_I	$I \to I$	$f(x) = \tilde{} x,$	bitwise complement
GrB_BinaryOp	GrB_LOR	`bool` $\times$ `bool` $\to$ `bool`	$f(x, y) = x \vee y,$	logical OR
GrB_BinaryOp	GrB_LAND	`bool` $\times$ `bool` $\to$ `bool`	$f(x, y) = x \wedge y,$	logical AND
GrB_BinaryOp	GrB_LXOR	`bool` $\times$ `bool` $\to$ `bool`	$f(x, y) = x \oplus y,$	logical XOR
GrB_BinaryOp	GrB_LXNOR	`bool` $\times$ `bool` $\to$ `bool`	$f(x, y) = \overline{x \oplus y},$	logical XNOR
GrB_BinaryOp	GrB_BOR_I	$I \times I \to I$	$f(x, y) = x \mid y,$	bitwise OR
GrB_BinaryOp	GrB_BAND_I	$I \times I \to I$	$f(x, y) = x \,\&\, y,$	bitwise AND
GrB_BinaryOp	GrB_BXOR_I	$I \times I \to I$	$f(x, y) = x \;\hat{}\; y,$	bitwise XOR
GrB_BinaryOp	GrB_BXNOR_I	$I \times I \to I$	$f(x, y) = \overline{x \;\hat{}\; y},$	bitwise XNOR
GrB_BinaryOp	GrB_EQ_T	$T \times T \to$ `bool`	$f(x, y) = (x == y)$	equal
GrB_BinaryOp	GrB_NE_T	$T \times T \to$ `bool`	$f(x, y) = (x \neq y)$	not equal
GrB_BinaryOp	GrB_GT_T	$T \times T \to$ `bool`	$f(x, y) = (x > y)$	greater than
GrB_BinaryOp	GrB_LT_T	$T \times T \to$ `bool`	$f(x, y) = (x < y)$	less than
GrB_BinaryOp	GrB_GE_T	$T \times T \to$ `bool`	$f(x, y) = (x \geq y)$	greater than or equal
GrB_BinaryOp	GrB_LE_T	$T \times T \to$ `bool`	$f(x, y) = (x \leq y)$	less than or equal
GrB_BinaryOp	GrB_ONEB_T	$T \times T \to T$	$f(x, y) = 1,$	1 (cast to T)
GrB_BinaryOp	GrB_FIRST_T	$T \times T \to T$	$f(x, y) = x,$	first argument
GrB_BinaryOp	GrB_SECOND_T	$T \times T \to T$	$f(x, y) = y,$	second argument
GrB_BinaryOp	GrB_MIN_T	$T \times T \to T$	$f(x, y) = (x < y)\ ?\ x : y,$	minimum
GrB_BinaryOp	GrB_MAX_T	$T \times T \to T$	$f(x, y) = (x > y)\ ?\ x : y,$	maximum
GrB_BinaryOp	GrB_PLUS_T	$T \times T \to T$	$f(x, y) = x + y,$	addition
GrB_BinaryOp	GrB_MINUS_T	$T \times T \to T$	$f(x, y) = x - y,$	subtraction
GrB_BinaryOp	GrB_TIMES_T	$T \times T \to T$	$f(x, y) = xy,$	multiplication
GrB_BinaryOp	GrB_DIV_T	$T \times T \to T$	$f(x, y) = \frac{x}{y},$	division

Table 3.6: Predefined index unary operators for GraphBLAS in C. The T can be any suffix from Table 3.2. I_{U64} refers to the unsigned 64-bit, GrB_Index, integer type, I_{32} refers to the signed, 32-bit integer type, and I_{64} refers to signed, 64-bit integer type. The parameters, u_i or A_{ij}, are the stored values from the containers where the i and j parameters are set to the row and column indices corresponding to the location of the stored value. When operating on vectors, j will be passed with a zero value. Finally, s is an additional scalar value used in the operators. The expressions in the "Description" column are to be treated as mathematical specifications. That is, for the index arithmetic functions in the first two groups below, each one of i, j, and s is interpreted as an integer number in the set $\mathbb{Z}$. Functions are evaluated using arithmetic in $\mathbb{Z}$, producing a result value that is also in $\mathbb{Z}$. The result value is converted to the output type according to the rules of the C language. In particular, if the value cannot be represented as a signed 32- or 64-bit integer type, the output is implementation defined. Any deviations from this ideal behavior, including limitations on the values of i, j, and s, or possible overflow and underflow conditions, must be defined by the implementation.

Operator type Type	GraphBLAS Name	Domains (− is don't care) A, u	i, j	s	result	Description			
GrB_IndexUnaryOp	GrB_ROWINDEX_$I_{32/64}$	−	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	$(i + s)$,	replace with its row index (+ s)
		−	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(u_i, i, 0, s)$	=	$(i + s)$	
GrB_IndexUnaryOp	GrB_COLINDEX_$I_{32/64}$	−	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	$(j + s)$	replace with its column index (+ s)
GrB_IndexUnaryOp	GrB_DIAGINDEX_$I_{32/64}$	−	I_{U64}	$I_{32/64}$	$I_{32/64}$	$f(A_{ij}, i, j, s)$	=	$(j - i + s)$	replace with its diagonal index (+ s)
GrB_IndexUnaryOp	GrB_TRIL	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j \leq i + s)$	triangle on or below diagonal s
GrB_IndexUnaryOp	GrB_TRIU	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j \geq i + s)$	triangle on or above diagonal s
GrB_IndexUnaryOp	GrB_DIAG	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j == i + s)$	diagonal s
GrB_IndexUnaryOp	GrB_OFFDIAG	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j \neq i + s)$	all but diagonal s
GrB_IndexUnaryOp	GrB_COLLE	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j \leq s)$	columns less or equal to s
GrB_IndexUnaryOp	GrB_COLGT	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(j > s)$	columns greater than s
GrB_IndexUnaryOp	GrB_ROWLE	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(i \leq s)$,	rows less or equal to s
		−	I_{U64}	I_{64}	bool	$f(u_i, i, 0, s)$	=	$(i \leq s)$	
GrB_IndexUnaryOp	GrB_ROWGT	−	I_{U64}	I_{64}	bool	$f(A_{ij}, i, j, s)$	=	$(i > s)$,	rows greater than s
		−	I_{U64}	I_{64}	bool	$f(u_i, i, 0, s)$	=	$(i > s)$	
GrB_IndexUnaryOp	GrB_VALUEEQ_T	T	−	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} == s)$,	elements equal to value s
		T	−	T	bool	$f(u_i, i, 0, s)$	=	$(u_i == s)$	
GrB_IndexUnaryOp	GrB_VALUENE_T	T	−	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} \neq s)$,	elements not equal to value s
		T	−	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \neq s)$	
GrB_IndexUnaryOp	GrB_VALUELT_T	T	−	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} < s)$,	elements less than value s
		T	−	T	bool	$f(u_i, i, 0, s)$	=	$(u_i < s)$	
GrB_IndexUnaryOp	GrB_VALUELE_T	T	−	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} \leq s)$,	elements less or equal to value s
		T	−	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \leq s)$	
GrB_IndexUnaryOp	GrB_VALUEGT_T	T	−	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} > s)$,	elements greater than value s
		T	−	T	bool	$f(u_i, i, 0, s)$	=	$(u_i > s)$	
GrB_IndexUnaryOp	GrB_VALUEGE_T	T	−	T	bool	$f(A_{ij}, i, j, s)$	=	$(A_{ij} \geq s)$,	elements greater or equal to value s
		T	−	T	bool	$f(u_i, i, 0, s)$	=	$(u_i \geq s)$	

3.4.2 Monoids

A GraphBLAS *monoid* $M = \langle D, \odot, 0\rangle$ is defined by a single domain D, an *associative*[1] operation $\odot : D \times D \rightarrow D$, and an identity element $0 \in D$. For a given GraphBLAS monoid $M = \langle D, \odot, 0\rangle$ we define $\mathbf{D}(M) = D$, $\bigodot(M) = \odot$, and $\mathbf{0}(M) = 0$. A GraphBLAS monoid is equivalent to the conventional *monoid* algebraic structure.

Let $F = \langle D, D, D, \odot\rangle$ be an associative GraphBLAS binary operator with identity element $0 \in D$. Then $M = \langle F, 0\rangle = \langle D, \odot, 0\rangle$ is a GraphBLAS monoid. If $\odot$ is commutative, then M is said to be a *commutative monoid*. If a monoid M is created using an operator $\odot$ that is not associative, the outcome of GraphBLAS operations using such a monoid is undefined.

User-defined monoids can be created with calls to GrB_Monoid_new (see Section 4.2.1). The GraphBLAS C API predefines a number of monoids that are listed in Table 3.7. Predefined monoids are named GrB_*op*_MONOID_T, where *op* is the name of the predefined GraphBLAS operator used as the associative binary operation of the monoid and T is the domain (type) of the monoid.

3.4.3 Semirings

A GraphBLAS *semiring* $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0\rangle$ is defined by three domains D_{out}, D_{in_1}, and D_{in_2}; an *associative*[1] and commutative additive operation $\oplus : D_{out} \times D_{out} \rightarrow D_{out}$; a multiplicative operation $\otimes : D_{in_1} \times D_{in_2} \rightarrow D_{out}$; and an identity element $0 \in D_{out}$. For a given GraphBLAS semiring $S = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0\rangle$ we define $\mathbf{D}_{in_1}(S) = D_{in_1}$, $\mathbf{D}_{in_2}(S) = D_{in_2}$, $\mathbf{D}_{out}(S) = D_{out}$, $\bigoplus(S) = \oplus$, $\bigotimes(S) = \otimes$, and $\mathbf{0}(S) = 0$.

Let $F = \langle D_{out}, D_{in_1}, D_{in_2}, \otimes\rangle$ be an operator and let $A = \langle D_{out}, \oplus, 0\rangle$ be a commutative monoid, then $S = \langle A, F\rangle = \langle D_{out}, D_{in_1}, D_{in_2}, \oplus, \otimes, 0\rangle$ is a semiring.

In a GraphBLAS semiring, the multiplicative operator does not have to distribute over the additive operator. This is unlike the conventional *semiring* algebraic structure.

Note: There must be one GraphBLAS monoid in every semiring which serves as the semiring's additive operator and specifies the same domain for its inputs and output parameters. If this monoid is not a commutative monoid, the outcome of GraphBLAS operations using the semiring is undefined.

A UML diagram of the conceptual hierarchy of object classes in GraphBLAS algebra (binary operators, monoids, and semirings) is shown in Figure 3.1.

User-defined semirings can be created with calls to GrB_Semiring_new (see Section 4.2.1). A list of predefined true semirings and convenience semirings can be found in Tables 3.8 and 3.9, respectively. Predefined semirings are named GrB_*add*_*mul*_SEMIRING_T, where *add* is the semiring additive operation, *mul* is the semiring multiplicative operation and T is the domain (type) of the semiring.

[1] It is expected that implementations of the GraphBLAS will utilize floating point arithmetic such as that defined in the IEEE-754 standard even though floating point arithmetic is not strictly associative.

Table 3.7: Predefined monoids for GraphBLAS in C. Maximum and minimum values for the various integral types are defined in `stdint.h`. Floating-point infinities are defined in `math.h`. The x in UINTx or INTx can be one of 8, 16, 32, or 64; whereas in FPx, it can be 32 or 64.

GraphBLAS identifier	Domains, T ($T \times T \rightarrow T$)	Identity	Description
GrB_PLUS_MONOID_T	UINTx	0	addition
	INTx	0	
	FPx	0	
GrB_TIMES_MONOID_T	UINTx	1	multiplication
	INTx	1	
	FPx	1	
GrB_MIN_MONOID_T	UINTx	UINTx_MAX	minimum
	INTx	INTx_MAX	
	FPx	INFINITY	
GrB_MAX_MONOID_T	UINTx	0	maximum
	INTx	INTx_MIN	
	FPx	-INFINITY	
GrB_LOR_MONOID_BOOL	BOOL	false	logical OR
GrB_LAND_MONOID_BOOL	BOOL	true	logical AND
GrB_LXOR_MONOID_BOOL	BOOL	false	logical XOR (not equal)
GrB_LXNOR_MONOID_BOOL	BOOL	true	logical XNOR (equal)

Table 3.8: Predefined true semirings for GraphBLAS in C where the additive identity is the multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

GraphBLAS identifier	Domains, T ($T \times T \rightarrow T$)	$+$ identity $\times$ annihilator	Description
GrB_PLUS_TIMES_SEMIRING_T	UINTx	0	arithmetic semiring
	INTx	0	
	FPx	0	
GrB_MIN_PLUS_SEMIRING_T	UINTx	UINTx_MAX	min-plus semiring
	INTx	INTx_MAX	
	FPx	INFINITY	
GrB_MAX_PLUS_SEMIRING_T	INTx	INTx_MIN	max-plus semiring
	FPx	-INFINITY	
GrB_MIN_TIMES_SEMIRING_T	UINTx	UINTx_MAX	min-times semiring
GrB_MIN_MAX_SEMIRING_T	UINTx	UINTx_MAX	min-max semiring
	INTx	INTx_MAX	
	FPx	INFINITY	
GrB_MAX_MIN_SEMIRING_T	UINTx	0	max-min semiring
	INTx	INTx_MIN	
	FPx	-INFINITY	
GrB_MAX_TIMES_SEMIRING_T	UINTx	0	max-times semiring
GrB_PLUS_MIN_SEMIRING_T	UINTx	0	plus-min semiring
GrB_LOR_LAND_SEMIRING_BOOL	BOOL	false	Logical semiring
GrB_LAND_LOR_SEMIRING_BOOL	BOOL	true	"and-or" semiring
GrB_LXOR_LAND_SEMIRING_BOOL	BOOL	false	same as NE_LAND
GrB_LXNOR_LOR_SEMIRING_BOOL	BOOL	true	same as EQ_LOR

Table 3.9: Other useful predefined semirings for GraphBLAS in C that don't have a multiplicative annihilator. The x can be one of 8, 16, 32, or 64 in UINTx or INTx, and can be 32 or 64 in FPx.

GraphBLAS identifier	Domains, T ($T \times T \rightarrow T$)	+ identity	Description
GrB_MAX_PLUS_SEMIRING_T	UINTx	0	max-plus semiring
GrB_MIN_TIMES_SEMIRING_T	INTx	INTx_MAX	min-times semiring
	FPx	INFINITY	
GrB_MAX_TIMES_SEMIRING_T	INTx	INTx_MIN	max-times semiring
	FPx	-INFINITY	
GrB_PLUS_MIN_SEMIRING_T	INTx	0	plus-min semiring
	FPx	0	
GrB_MIN_FIRST_SEMIRING_T	UINTx	UINTx_MAX	min-select first semiring
	INTx	INTx_MAX	
	FPx	INFINITY	
GrB_MIN_SECOND_SEMIRING_T	UINTx	UINTx_MAX	min-select second semiring
	INTx	INTx_MAX	
	FPx	INFINITY	
GrB_MAX_FIRST_SEMIRING_T	UINTx	0	max-select first semiring
	INTx	INTx_MIN	
	FPx	-INFINITY	
GrB_MAX_SECOND_SEMIRING_T	UINTx	0	max-select second semiring
	INTx	INTx_MIN	
	FPx	-INFINITY	

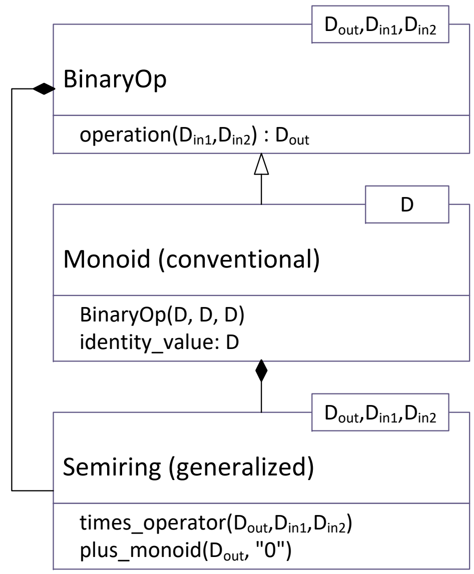

Figure 3.1: Hierarchy of algebraic object classes in GraphBLAS. GraphBLAS semirings consist of a conventional monoid with one domain for the addition function, and a binary operator with three domains for the multiplication function.

3.5 Collections

3.5.1 Scalars

A *GraphBLAS scalar*, $s = \langle D, \{\sigma\}\rangle$, is defined by a domain D, and a set of zero or one *scalar value*, σ, where $\sigma \in D$. We define $\textbf{size}(s) = 1$ (constant), and $\mathbf{L}(s) = \{\sigma\}$. The set $\mathbf{L}(s)$ is called the *contents* of the GraphBLAS scalar s. We also define $\mathbf{D}(s) = D$. Finally, $\textbf{val}(s)$ is a reference to the scalar value, σ, if the GraphBLAS scalar is not empty, and is undefined otherwise.

3.5.2 Vectors

A vector $\mathbf{v} = \langle D, N, \{(i, v_i)\}\rangle$ is defined by a domain D, a size $N > 0$, and a set of tuples (i, v_i) where $0 \leq i < N$ and $v_i \in D$. A particular value of i can appear at most once in $\mathbf{v}$. We define $\textbf{size}(\mathbf{v}) = N$ and $\mathbf{L}(\mathbf{v}) = \{(i, v_i)\}$. The set $\mathbf{L}(\mathbf{v})$ is called the *content* of vector $\mathbf{v}$. We also define the set $\textbf{ind}(\mathbf{v}) = \{i : (i, v_i) \in \mathbf{L}(\mathbf{v})\}$ (called the *structure* of $\mathbf{v}$), and $\mathbf{D}(\mathbf{v}) = D$. For a vector $\mathbf{v}$, $\mathbf{v}(i)$ is a reference to v_i if $(i, v_i) \in \mathbf{L}(\mathbf{v})$ and is undefined otherwise.

3.5.3 Matrices

A matrix $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\}\rangle$ is defined by a domain D, its number of rows $M > 0$, its number of columns $N > 0$, and a set of tuples (i, j, A_{ij}) where $0 \leq i < M$, $0 \leq j < N$, and $A_{ij} \in D$. A particular pair of values i, j can appear at most once in $\mathbf{A}$. We define $\mathbf{ncols}(\mathbf{A}) = N$, $\mathbf{nrows}(\mathbf{A}) = M$, and $\mathbf{L}(\mathbf{A}) = \{(i, j, A_{ij})\}$. The set $\mathbf{L}(\mathbf{A})$ is called the *content* of matrix $\mathbf{A}$. We also define the sets $\mathbf{indrow}(\mathbf{A}) = \{i : \exists(i, j, A_{ij}) \in \mathbf{A}\}$ and $\mathbf{indcol}(\mathbf{A}) = \{j : \exists(i, j, A_{ij}) \in \mathbf{A}\}$. (These are the sets of nonempty rows and columns of $\mathbf{A}$, respectively.) The *structure* of matrix $\mathbf{A}$ is the set $\mathbf{ind}(\mathbf{A}) = \{(i, j) : (i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})\}$, and $\mathbf{D}(\mathbf{A}) = D$. For a matrix $\mathbf{A}$, $\mathbf{A}(i, j)$ is a reference to A_{ij} if $(i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})$ and is undefined otherwise.

If $\mathbf{A}$ is a matrix and $0 \leq j < N$, then $\mathbf{A}(:, j) = \langle D, M, \{(i, A_{ij}) : (i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})\}\rangle$ is a vector called the j-th *column* of $\mathbf{A}$. Correspondingly, if $\mathbf{A}$ is a matrix and $0 \leq i < M$, then $\mathbf{A}(i, :) = \langle D, N, \{(j, A_{ij}) : (i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})\}\rangle$ is a vector called the i-th *row* of $\mathbf{A}$.

Given a matrix $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\}\rangle$, its *transpose* is another matrix $\mathbf{A}^T = \langle D, N, M, \{(j, i, A_{ij}) : (i, j, A_{ij}) \in \mathbf{L}(\mathbf{A})\}\rangle$.

3.5.3.1 External matrix formats

The specification also supports the export and import of matrices to/from a number of commonly used formats, such as COO, CSR, and CSC formats. When importing or exporting a matrix to or from a GraphBLAS object using GrB_Matrix_import (§ 4.2.4.17) or GrB_Matrix_export (§ 4.2.4.16), it is necessary to specify the data format for the matrix data external to GraphBLAS, which is being imported from or exported to. This non-opaque data format is specified using an argument of enumeration type GrB_Format that is used to indicate one of a number of predefined formats. The predefined values of GrB_Format are specified in Table 3.10. A precise definition of the non-opaque data formats can be found in Appendix B.

Table 3.10: GrB_Format enumeration literals and corresponding values for matrix import and export methods.

Symbol	Value	Description
GrB_CSR_FORMAT	0	Specifies the compressed sparse row matrix format.
GrB_CSC_FORMAT	1	Specifies the compressed sparse column matrix format.
GrB_COO_FORMAT	2	Specifies the sparse coordinate matrix format.

3.5.4 Masks

The GraphBLAS C API defines an opaque object called a *mask*. The mask is used to control how computed values are stored in the output from a method. The mask is an *internal* opaque object; that is, it is never exposed as a variable within an application.

The mask is formed from input objects to the method that uses the mask. For example, a GraphBLAS method may be called with a matrix as the mask parameter. The internal mask object is constructed from the input matrix in one of two ways. In the default case, an element of the mask is created for each tuple that exists in the matrix for which the value of the tuple cast to Boolean evaluates to `true`. Alternatively, the user can specify *structure*-only behavior where an element of the mask is created for each tuple that exists in the matrix *regardless* of the value stored in the input matrix.

The internal mask object can be either a one- or a two-dimensional construct. One- and two-dimensional masks, described more formally below, are similar to vectors and matrices, respectively, except that they have structure (indices) but no values. When needed, a value is implied for the elements of a mask with an implied value of `true` for elements that exist and an implied value of `false` for elements that do not exist (i.e., the locations of the mask that do not have a stored value imply a value of `false`). Hence, even though a mask does not contain any values, it can be considered to imply values from a Boolean domain.

A one-dimensional mask $\mathbf{m} = \langle N, \{i\}\rangle$ is defined by its number of elements $N > 0$, and a set $\mathbf{ind}(\mathbf{m})$ of indices $\{i\}$ where $0 \leq i < N$. A particular value of i can appear at most once in $\mathbf{m}$. We define $\mathbf{size}(\mathbf{m}) = N$. The set $\mathbf{ind}(\mathbf{m})$ is called the *structure* of mask $\mathbf{m}$.

A two-dimensional mask $\mathbf{M} = \langle M, N, \{(i, j)\}\rangle$ is defined by its number of rows $M > 0$, its number of columns $N > 0$, and a set $\mathbf{ind}(\mathbf{M})$ of tuples (i, j) where $0 \leq i < M$, $0 \leq j < N$. A particular pair of values i, j can appear at most once in $\mathbf{M}$. We define $\mathbf{ncols}(\mathbf{M}) = N$, and $\mathbf{nrows}(\mathbf{M}) = M$. We also define the sets $\mathbf{indrow}(\mathbf{M}) = \{i : \exists(i, j) \in \mathbf{ind}(\mathbf{M})\}$ and $\mathbf{indcol}(\mathbf{M}) = \{j : \exists(i, j) \in \mathbf{ind}(\mathbf{M})\}$. These are the sets of nonempty rows and columns of $\mathbf{M}$, respectively. The set $\mathbf{ind}(\mathbf{M})$ is called the *structure* of mask $\mathbf{M}$.

One common operation on masks is the *complement*. For a one-dimensional mask $\mathbf{m}$ this is denoted as $\neg\mathbf{m}$. For a two-dimensional mask $\mathbf{M}$, this is denoted as $\neg\mathbf{M}$. The complement of a one-dimensional mask $\mathbf{m}$ is defined as $\mathbf{ind}(\neg\mathbf{m}) = \{i : 0 \leq i < N, i \notin \mathbf{ind}(\mathbf{m})\}$. It is the set of all possible indices that do not appear in $\mathbf{m}$. The complement of a two-dimensional mask $\mathbf{M}$ is defined as the set $\mathbf{ind}(\neg\mathbf{M}) = \{(i, j) : 0 \leq i < M, 0 \leq j < N, (i, j) \notin \mathbf{ind}(\mathbf{M})\}$. It is the set of all possible indices that do not appear in $\mathbf{M}$.

3.6 Descriptors

Descriptors are used to modify the behavior of a GraphBLAS method. When present in the signature of a method, they appear as the last argument in the method. Descriptors specify how the other input arguments corresponding to GraphBLAS collections – vectors, matrices, and masks – should be processed (modified) before the main operation of a method is performed. A complete list of what descriptors are capable of are presented in this section.

The descriptor is a lightweight object. It is composed of (*field*, *value*) pairs where the *field* selects one of the GraphBLAS objects from the argument list of a method and the *value* defines the indicated modification associated with that object. For example, a descriptor may specify that a particular input matrix needs to be transposed or that a mask needs to be complemented (defined in Section 3.5.4) before using it in the operation.

For the purpose of constructing descriptors, the arguments of a method that can be modified are identified by specific field names. The output parameter (typically the first parameter in a GraphBLAS method) is indicated by the field name, GrB_OUTP. The mask is indicated by the GrB_MASK field name. The input parameters corresponding to the input vectors and matrices are indicated by GrB_INP0 and GrB_INP1 in the order they appear in the signature of the GraphBLAS method. The descriptor is an opaque object and hence we do not define how objects of this type should be implemented. When referring to (*field*, *value*) pairs for a descriptor, however, we often use the informal notation desc[GrB_Desc_Field].GrB_Desc_Value without implying that a descriptor is to be implemented as an array of structures (in fact, field values can be used in conjunction with multiple values that are composable). We summarize all types, field names, and values used with descriptors in Table 3.11.

In the definitions of the GraphBLAS methods, we often refer to the *default behavior* of a method with respect to the action of a descriptor. If a descriptor is not provided or if the value associated with a particular field in a descriptor is not set, the default behavior of a GraphBLAS method is defined as follows:

- Input matrices are not transposed.
- The mask is used, as is, without complementing, and stored values are examined to determine whether they evaluate to `true` or `false`.
- Values of the output object that are not directly modified by the operation are preserved.

GraphBLAS specifies all of the valid combinations of (field, value) pairs as predefined descriptors. Their identifiers and the corresponding set of (field, value) pairs for that identfier are shown in Table 3.12.

3.7 GrB_Info return values

All GraphBLAS methods return a GrB_Info enumeration value. The three types of return codes (informational, API error, and execution error) and their corresponding values are listed in Table 3.13.

Table 3.11: Descriptors are GraphBLAS objects passed as arguments to GraphBLAS operations to modify other GraphBLAS objects in the operation's argument list. A descriptor, desc, has one or more (*field*, *value*) pairs indicated as desc[GrB_Desc_Field].GrB_Desc_Value. In this table, we define all types and literals used with descriptors.

(a) Types used with GraphBLAS descriptors.

Type	Description
GrB_Descriptor	Type of a GraphBLAS descriptor object.
GrB_Desc_Field	The descriptor field enumeration.
GrB_Desc_Value	The descriptor value enumeration.

(b) Descriptor field names of type GrB_Desc_Field enumeration and corresponding values.

Field Name	Value	Description
GrB_OUTP	0	Field name for the output GraphBLAS object.
GrB_MASK	1	Field name for the mask GraphBLAS object.
GrB_INP0	2	Field name for the first input GraphBLAS object.
GrB_INP1	3	Field name for the second input GraphBLAS object.

(c) Descriptor field values of type GrB_Desc_Value enumeration and corresponding values.

Value Name	Value	Description
(reserved)	0	Unused
GrB_REPLACE	1	Clear the output object before assigning computed values.
GrB_COMP	2	Use the complement of the associated object. When combined with GrB_STRUCTURE, the complement of the structure of the associated object is used without evaluating the values stored.
GrB_TRAN	3	Use the transpose of the associated object.
GrB_STRUCTURE	4	The write mask is constructed from the structure (pattern of stored values) of the associated object. The stored values are not examined.

Table 3.12: Predefined GraphBLAS descriptors. The list includes all possible descriptors, according to the current standard. Columns list the possible fields and entries list the value(s) associated with those fields for a given descriptor.

Identifier	GrB_OUTP	GrB_MASK	GrB_INP0	GrB_INP1
GrB_NULL	–	–	–	–
GrB_DESC_T1	–	–	–	GrB_TRAN
GrB_DESC_T0	–	–	GrB_TRAN	–
GrB_DESC_T0T1	–	–	GrB_TRAN	GrB_TRAN
GrB_DESC_C	–	GrB_COMP	–	–
GrB_DESC_S	–	GrB_STRUCTURE	–	–
GrB_DESC_CT1	–	GrB_COMP	–	GrB_TRAN
GrB_DESC_ST1	–	GrB_STRUCTURE	–	GrB_TRAN
GrB_DESC_CT0	–	GrB_COMP	GrB_TRAN	–
GrB_DESC_ST0	–	GrB_STRUCTURE	GrB_TRAN	–
GrB_DESC_CT0T1	–	GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_ST0T1	–	GrB_STRUCTURE	GrB_TRAN	GrB_TRAN
GrB_DESC_SC	–	GrB_STRUCTURE, GrB_COMP	–	–
GrB_DESC_SCT1	–	GrB_STRUCTURE, GrB_COMP	–	GrB_TRAN
GrB_DESC_SCT0	–	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	–
GrB_DESC_SCT0T1	–	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_R	GrB_REPLACE	–	–	–
GrB_DESC_RT1	GrB_REPLACE	–	–	GrB_TRAN
GrB_DESC_RT0	GrB_REPLACE	–	GrB_TRAN	–
GrB_DESC_RT0T1	GrB_REPLACE	–	GrB_TRAN	GrB_TRAN
GrB_DESC_RC	GrB_REPLACE	GrB_COMP	–	–
GrB_DESC_RS	GrB_REPLACE	GrB_STRUCTURE	–	–
GrB_DESC_RCT1	GrB_REPLACE	GrB_COMP	–	GrB_TRAN
GrB_DESC_RST1	GrB_REPLACE	GrB_STRUCTURE	–	GrB_TRAN
GrB_DESC_RCT0	GrB_REPLACE	GrB_COMP	GrB_TRAN	–
GrB_DESC_RST0	GrB_REPLACE	GrB_STRUCTURE	GrB_TRAN	–
GrB_DESC_RCT0T1	GrB_REPLACE	GrB_COMP	GrB_TRAN	GrB_TRAN
GrB_DESC_RST0T1	GrB_REPLACE	GrB_STRUCTURE	GrB_TRAN	GrB_TRAN
GrB_DESC_RSC	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	–	–
GrB_DESC_RSCT1	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	–	GrB_TRAN
GrB_DESC_RSCT0	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	–
GrB_DESC_RSCT0T1	GrB_REPLACE	GrB_STRUCTURE, GrB_COMP	GrB_TRAN	GrB_TRAN

Table 3.13: Enumeration literals and corresponding values returned by GraphBLAS methods and operations.

(a) Informational return values

Symbol	Value	Description
GrB_SUCCESS	0	The method/operation completed successfully (blocking mode), or encountered no API errors (non-blocking mode).
GrB_NO_VALUE	1	A location in a matrix or vector is being accessed that has no stored value at the specified location.

(b) API errors

Symbol	Value	Description
GrB_UNINITIALIZED_OBJECT	-1	A GraphBLAS object is passed to a method before new was called on it.
GrB_NULL_POINTER	-2	A NULL is passed for a pointer parameter.
GrB_INVALID_VALUE	-3	Miscellaneous incorrect values.
GrB_INVALID_INDEX	-4	Indices passed are larger than dimensions of the matrix or vector being accessed.
GrB_DOMAIN_MISMATCH	-5	A mismatch between domains of collections and operations when user-defined domains are in use.
GrB_DIMENSION_MISMATCH	-6	Operations on matrices and vectors with incompatible dimensions.
GrB_OUTPUT_NOT_EMPTY	-7	An attempt was made to build a matrix or vector using an output object that already contains valid tuples (elements).
GrB_NOT_IMPLEMENTED	-8	An attempt was made to call a GraphBLAS method for a combination of input parameters that is not supported by a particular implementation.

(c) Execution errors

Symbol	Value	Description
GrB_PANIC	-101	Unknown internal error.
GrB_OUT_OF_MEMORY	-102	Not enough memory for operations.
GrB_INSUFFICIENT_SPACE	-103	The array provided is not large enough to hold output.
GrB_INVALID_OBJECT	-104	One of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error.
GrB_INDEX_OUT_OF_BOUNDS	-105	Reference to a vector or matrix element that is outside the defined dimensions of the object.
GrB_EMPTY_OBJECT	-106	One of the opaque GraphBLAS objects does not have a stored value.

Chapter 4

Methods

This chapter defines the behavior of all the methods in the GraphBLAS C API. All methods can be declared for use in programs by including the `GraphBLAS.h` header file.

We would like to emphasize that no GraphBLAS method will imply a predefined order over any associative operators. Implementations of the GraphBLAS are encouraged to exploit associativity to optimize performance of any GraphBLAS method. This holds even if the definition of the GraphBLAS method implies a fixed order for the associative operations.

4.1 Context methods

The methods in this section set up and tear down the GraphBLAS context within which all GraphBLAS methods must be executed. The initialization of this context also includes the specification of which execution mode is to be used.

4.1.1 init: Initialize a GraphBLAS context

Creates and initializes a GraphBLAS C API context.

C Syntax

```
GrB_Info GrB_init(GrB_Mode mode);
```

Parameters

mode Mode for the GraphBLAS context. Must be either GrB_BLOCKING or GrB_NONBLOCKING.

Return Values

GrB_SUCCESS	operation completed successfully.
GrB_PANIC	unknown internal error.
GrB_INVALID_VALUE	invalid mode specified, or method called multiple times.

Description

The init method creates and initializes a GraphBLAS C API context. The argument to GrB_init defines the mode for the context. The two available modes are:

- GrB_BLOCKING: In this mode, each method in a sequence returns after its computations have completed and output arguments are available to subsequent statements in an application. When executing in GrB_BLOCKING mode, the methods execute in program order.
- GrB_NONBLOCKING: In this mode, methods in a sequence may return after arguments in the method have been tested for dimension and domain compatibility within the method but potentially before their computations complete. Output arguments are available to subsequent GraphBLAS methods in an application. When executing in GrB_NONBLOCKING mode, the methods in a sequence may execute in any order that preserves the mathematical result defined by the sequence.

An application can only create one context per execution instance. An application may only call GrB_Init once. Calling GrB_Init more than once results in undefined behavior.

4.1.2 finalize: Finalize a GraphBLAS context

Terminates and frees any internal resources created to support the GraphBLAS C API context.

C Syntax

```
GrB_Info GrB_finalize();
```

Return Values

GrB_SUCCESS	operation completed successfully.
GrB_PANIC	unknown internal error.

Description

The finalize method terminates and frees any internal resources created to support the GraphBLAS C API context. GrB_finalize may only be called after a context has been initialized by calling GrB_init, or else undefined behavior occurs. After GrB_finalize has been called to finalize a GraphBLAS context, calls to any GraphBLAS methods, including GrB_finalize, will result in undefined behavior.

4.1.3 getVersion: Get the version number of the standard.

Query the library for the version number of the standard that this library implements.

C Syntax

```
GrB_Info GrB_getVersion(unsigned int *version,
                        unsigned int *subversion);
```

Parameters

version (OUT) On successful return will hold the value of the major version number.

version (OUT) On successful return will hold the value of the subversion number.

Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

Description

The getVersion method is used to query the major and minor version number of the GraphBLAS C API specification that the library implements at runtime. To support compile time queries the following two macros shall also be defined by the library.

```
#define GRB_VERSION     2
#define GRB_SUBVERSION  0
```

4.2 Object methods

This section describes methods that setup and operate on GraphBLAS opaque objects but are not part of the the GraphBLAS math specification.

4.2.1 Algebra methods

4.2.1.1 Type_new: Construct a new GraphBLAS (user-defined) type

Creates a new user-defined GraphBLAS type. This type can then be used to create new operators, monoids, semirings, vectors and matrices.

C Syntax

```
GrB_Info GrB_Type_new(GrB_Type  *utype,
                      size_t     sizeof(ctype));
```

Parameters

utype (INOUT) On successful return, contains a handle to the newly created user-defined GraphBLAS type object.

ctype (IN) A C type that defines the new GraphBLAS user-defined type.

Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_NULL_POINTER utype pointer is NULL.

Description

Given a C type ctype, the Type_new method returns in utype a handle to a new GraphBLAS type that is equivalent to the C type. Variables of this ctype must be a struct, union, or fixed-size array. In particular, given two variables, src and dst, of type ctype, the following operation must be a valid way to copy the contents of src to dst:

```
memcpy(&dst, &src, sizeof(ctype))
```

A new, user-defined type utype should be destroyed with a call to GrB_free(utype) when no longer needed.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.1.2 UnaryOp_new: Construct a new GraphBLAS unary operator

Initializes a new GraphBLAS unary operator with a specified user-defined function and its types (domains).

C Syntax

```
GrB_Info GrB_UnaryOp_new(GrB_UnaryOp *unary_op,
                         void       (*unary_func)(void*, const void*),
                         GrB_Type     d_out,
                         GrB_Type     d_in);
```

Parameters

- unary_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS unary operator object.
- unary_func (IN) a pointer to a user-defined function that takes one input parameter of d_in's type and returns a value of d_out's type, both passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void *out, const void *in);
```

- d_out (IN) The GrB_Type of the return value of the unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
- d_in (IN) The GrB_Type of the input argument of the unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

Return Values

- GrB_SUCCESS operation completed successfully.
- GrB_PANIC unknown internal error.
- GrB_OUT_OF_MEMORY not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT any GrB_Type parameter (for user-defined types) has not been initialized by a call to GrB_Type_new.
- GrB_NULL_POINTER unary_op or unary_func pointers are NULL.

Description

The UnaryOp_new method creates a new GraphBLAS unary operator

$$f_u = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in}), \mathsf{unary\_func} \rangle$$

and returns a handle to it in unary_op.

The implementation of unary_func must be such that it works even if the d_out and d_in arguments are aliased. In other words, for all invocations of the function:

```
unary_func(out,in);
```

the value of out must be the same as if the following code was executed:

```
D(d_in) *tmp = malloc(sizeof(D(d_in)));
memcpy(tmp,in,sizeof(D(d_in)));
unary_func(out,tmp);
free(tmp);
```

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.1.3 BinaryOp_new: Construct a new GraphBLAS binary operator

Initializes a new GraphBLAS binary operator with a specified user-defined function and its types (domains).

C Syntax

```
GrB_Info GrB_BinaryOp_new(GrB_BinaryOp *binary_op,
                          void        (*binary_func)(void*,
                                                     const void*,
                                                     const void*),
                          GrB_Type      d_out,
                          GrB_Type      d_in1,
                          GrB_Type      d_in2);
```

Parameters

binary_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS binary operator object.

binary_func (IN) A pointer to a user-defined function that takes two input parameters of types d_in1 and d_in2 and returns a value of type d_out, all passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void *out, const void *in1, const void *in2);
```

d_out (IN) The GrB_Type of the return value of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

d_in1 (IN) The GrB_Type of the left hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

d_in2 (IN) The GrB_Type of the right hand argument of the binary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT the GrB_Type (for user-defined types) has not been initialized by a call to GrB_Type_new.

GrB_NULL_POINTER binary_op or binary_func pointer is NULL.

Description

The BinaryOp_new methods creates a new GraphBLAS binary operator

$$f_b = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{binary\_func} \rangle$$

and returns a handle to it in binary_op.

The implementation of binary_func must be such that it works even if any of the d_out, d_in1, and d_in2 arguments are aliased to each other. In other words, for all invocations of the function:

```
binary_func(out,in1,in2);
```

the value of out must be the same as if the following code was executed:

```
D(d_in1) *tmp1 = malloc(sizeof(D(d_in1)));
D(d_in2) *tmp2 = malloc(sizeof(D(d_in2)));
memcpy(tmp1,in1,sizeof(D(d_in1)));
memcpy(tmp2,in2,sizeof(D(d_in2)));
binary_func(out,tmp1,tmp2);
free(tmp2);
free(tmp1);
```

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.1.4 Monoid_new: Construct a new GraphBLAS monoid

Creates a new monoid with specified binary operator and identity value.

C Syntax

```
GrB_Info GrB_Monoid_new(GrB_Monoid    *monoid,
                        GrB_BinaryOp   binary_op,
                        <type>         identity);
```

Parameters

- monoid (INOUT) On successful return, contains a handle to the newly created GraphBLAS monoid object.
- binary_op (IN) An existing GraphBLAS associative binary operator whose input and output types are the same.
- identity (IN) The value of the identity element of the monoid. Must be the same type as the type used by the binary_op operator.

Return Values

GrB_SUCCESS	operation completed successfully.
GrB_PANIC	unknown internal error.
GrB_OUT_OF_MEMORY	not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	the GrB_BinaryOp (for user-defined operators) has not been initialized by a call to GrB_BinaryOp_new.
GrB_NULL_POINTER	monoid pointer is NULL.
GrB_DOMAIN_MISMATCH	all three argument types of the binary operator and the type of the identity value are not the same.

Description

The Monoid_new method creates a new monoid $M = \langle \mathbf{D}(\text{binary\_op}), \text{binary\_op}, \text{identity} \rangle$ and returns a handle to it in monoid.

If binary_op is not associative, the results of GraphBLAS operations that require associativity of this monoid will be undefined.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.1.5 Semiring_new: Construct a new GraphBLAS semiring

Creates a new semiring with specified domain, operators, and elements.

C Syntax

```
GrB_Info GrB_Semiring_new(GrB_Semiring  *semiring,
                          GrB_Monoid     add_op,
                          GrB_BinaryOp   mul_op);
```

Parameters

- **semiring** (INOUT) On successful return, contains a handle to the newly created GraphBLAS semiring.
- **add_op** (IN) An existing GraphBLAS commutative monoid that specifies the addition operator and its identity.
- **mul_op** (IN) An existing GraphBLAS binary operator that specifies the semiring's multiplication operator. In addition, mul_op's output domain, $\mathbf{D}_{out}(\text{mul\_op})$, must be the same as the add_op's domain $\mathbf{D}(\text{add\_op})$.

Return Values

- **GrB_SUCCESS** operation completed successfully.
- **GrB_PANIC** unknown internal error.
- **GrB_OUT_OF_MEMORY** not enough memory available for this method to complete.
- **GrB_UNINITIALIZED_OBJECT** the add_op (for user-define monoids) object has not been initialized with a call to GrB_Monoid_new or the mul_op (for user-defined operators) object has not been not been initialized by a call to GrB_BinaryOp_new.

GrB_NULL_POINTER semiring pointer is NULL.

GrB_DOMAIN_MISMATCH the output domain of mul_op does not match the domain of the add_op monoid.

Description

The Semiring_new method creates a new semiring:

$$S = \langle \mathbf{D}_{out}(\mathsf{mul\_op}), \mathbf{D}_{in_1}(\mathsf{mul\_op}), \mathbf{D}_{in_2}(\mathsf{mul\_op}), \mathsf{add\_op}, \mathsf{mul\_op}, \mathbf{0}(\mathsf{add\_op}) \rangle$$

and returns a handle to it in semiring. Note that $\mathbf{D}_{out}(\mathsf{mul\_op})$ must be the same as $\mathbf{D}(\mathsf{add\_op})$.

If add_op is not commutative, then GraphBLAS operations using this semiring will be undefined.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.1.6 IndexUnaryOp_new: Construct a new GraphBLAS index unary operator

Initializes a new GraphBLAS index unary operator with a specified user-defined function and its types (domains).

C Syntax

```
GrB_Info GrB_IndexUnaryOp_new(GrB_IndexUnaryOp   *index_unary_op,
                              void (*index_unary_func)(void*,
                                                       const void*,
                                                       GrB_Index,
                                                       GrB_Index,
                                                       const void*),
                              GrB_Type            d_out,
                              GrB_Type            d_in1,
                              GrB_Type            d_in2);
```

Parameters

index_unary_op (INOUT) On successful return, contains a handle to the newly created GraphBLAS index unary operator object.

index_unary_func (IN) A pointer to a user-defined function that takes input parameters of types d_in1, GrB_Index, GrB_Index and d_in2 and returns a value of type d_out. Except for the GrB_Index parameters, all are passed as void pointers. Specifically the signature of the function is expected to be of the form:

```
void func(void       *out,
          const void *in1,
          GrB_Index   row_index,
          GrB_Index   col_index,
          const void *in2);
```

- **d_out** (IN) The GrB_Type of the return value of the index unary operator being created. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
- **d_in1** (IN) The GrB_Type of the first input argument of the index unary operator being created and corresponds to the stored values of the GrB_Vector or GrB_Matrix being operated on. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.
- **d_in2** (IN) The GrB_Type of the last input argument of the index unary operator being created and corresponds to a scalar provided by the GraphBLAS operation that uses this operator. Should be one of the predefined GraphBLAS types in Table 3.2, or a user-defined GraphBLAS type.

Return Values

- **GrB_SUCCESS** operation completed successfully.
- **GrB_PANIC** unknown internal error.
- **GrB_OUT_OF_MEMORY** not enough memory available for operation.
- **GrB_UNINITIALIZED_OBJECT** the GrB_Type (for user-defined types) has not been initialized by a call to GrB_Type_new.
- **GrB_NULL_POINTER** index_unary_op or index_unary_func pointer is NULL.

Description

The IndexUnaryOp_new methods creates a new GraphBLAS index unary operator

$$f_i = \langle \mathbf{D}(\mathsf{d\_out}), \mathbf{D}(\mathsf{d\_in1}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{GrB\_Index}), \mathbf{D}(\mathsf{d\_in2}), \mathsf{index\_unary\_func} \rangle$$

and returns a handle to it in index_unary_op.

The implementation of index_unary_func must be such that it works even if any of the d_out, d_in1, and d_in2 arguments are aliased to each other. In other words, for all invocations of the function:

```
index_unary_func(out,in1,row_index,col_index,n,in2);
```

the value of out must be the same as if the following code was executed (shown here for matrices):

```
GrB_Index row_index = ...;
GrB_Index col_index = ...;
D(d_in1) *tmp1 = malloc(sizeof(D(d_in1)));
D(d_in2) *tmp2 = malloc(sizeof(D(d_in2)));
memcpy(tmp1,in1,sizeof(D(d_in1)));
memcpy(tmp2,in2,sizeof(D(d_in2)));
index_unary_func(out,tmp1,row_index,col_index,tmp2);
free(tmp2);
free(tmp1);
```

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.2 Scalar methods

4.2.2.1 Scalar_new: Construct a new scalar

Creates a new empty scalar with specified domain.

C Syntax

```
GrB_Info GrB_Scalar_new(GrB_Scalar *s,
                        GrB_Type    d);
```

Parameters

- s (INOUT) On successful return, contains a handle to the newly created GraphBLAS scalar.
- d (IN) The type corresponding to the domain of the scalar being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar s is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).

GrB_NULL_POINTER The s pointer is NULL.

Description

Creates a new GraphBLAS scalar s of domain $\mathbf{D}(\mathsf{d})$ and empty $\mathbf{L}(s)$. The method returns a handle to the new scalar in s.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.2.2 Scalar_dup: Construct a copy of a GraphBLAS scalar

Creates a new scalar with the same domain and contents as another scalar.

C Syntax

```
GrB_Info GrB_Scalar_dup(GrB_Scalar       *t,
                        const GrB_Scalar  s);
```

Parameters

t (INOUT) On successful return, contains a handle to the newly created GraphBLAS scalar.

s (IN) The GraphBLAS scalar to be duplicated.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar t is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.

GrB_NULL_POINTER The t pointer is NULL.

Description

Creates a new scalar t of domain $\mathbf{D}(\mathsf{s})$ and contents $\mathbf{L}(\mathsf{s})$. The method returns a handle to the new scalar in t.

It is not an error to call this method more than once with the same output variable; however, the handle to the previously created object will be overwritten.

4.2.2.3 Scalar_clear: Clear/remove a stored value from a scalar

Removes the stored value from a scalar.

C Syntax

```
GrB_Info GrB_Scalar_clear(GrB_Scalar s);
```

Parameters

s (INOUT) An existing GraphBLAS scalar to clear.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output scalar s is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.

Description

Removes the stored value from an existing scalar. After the call, **L**(s) is empty. The size of the scalar does not change.

4.2.2.4 Scalar_nvals: Number of stored elements in a scalar

Retrieve the number of stored elements in a scalar (either zero or one).

C Syntax

```
GrB_Info GrB_Scalar_nvals(GrB_Index        *nvals,
                          const GrB_Scalar  s);
```

Parameters

nvals (OUT) On successful return, this is set to the number of stored elements in the scalar (zero or one).

s (IN) An existing GraphBLAS scalar being queried.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.

GrB_NULL_POINTER The nvals pointer is NULL.

Description

Return **nvals**(s) in nvals. This is the number of stored elements in scalar s, which is the size of **L**(s), and can only be either zero or one (see Section 3.5.1).

4.2.2.5 Scalar_setElement: Set the single element in a scalar

Set the single element of a scalar to a given value.

C Syntax

```
GrB_Info GrB_Scalar_setElement(GrB_Scalar   s,
                                <type>       val);
```

Parameters

- s (INOUT) An existing GraphBLAS scalar for which the element is to be assigned.
- val (IN) Scalar value to assign. The type must be compatible with the domain of s.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output scalar s is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.
- GrB_DOMAIN_MISMATCH The domains of s and val are incompatible.

Description

First, val and output GraphBLAS scalar are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}(\mathsf{s})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Scalar_setElement ends and the domain mismatch error listed above is returned.

We are now ready to carry out the assignment val; that is:

$$\mathsf{s}(0) = \mathsf{val}$$

If s already had a stored value, it will be overwritten; otherwise, the new value is stored in s.

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of s is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of scalar s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.2.6 Scalar_extractElement: Extract a single element from a scalar.

Assign a non-opaque scalar with the value of the element stored in a GraphBLAS scalar.

C Syntax

```
GrB_Info GrB_Scalar_extractElement(<type>           *val,
                                   const GrB_Scalar  s);
```

Parameters

- **val** (INOUT) Pointer to a non-opaque scalar of type that is compatible with the domain of scalar s. On successful return, val holds the result of the operation, and any previous value in val is overwritten.
- **s** (IN) The GraphBLAS scalar from which an element is extracted.

Return Values

- **GrB_SUCCESS** In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val, has been computed and is ready to be used in the next method of the sequence.
- **GrB_PANIC** Unknown internal error.

GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	The GraphBLAS scalar, s, has not been initialized by a call to Scalar_new or Scalar_dup.
GrB_NULL_POINTER	val pointer is NULL.
GrB_DOMAIN_MISMATCH	The domains of the scalar or scalar are incompatible.
GrB_NO_VALUE	There is no stored value in the scalar.

Description

First, val and input GraphBLAS scalar are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}(\mathsf{s})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Scalar_extractElement ends and the domain mismatch error listed above is returned.

Then, if no value is currently stored in the GraphBLAS scalar, the method returns GrB_NO_VALUE and val remains unchanged.

Finally the extract into the output argument, val can be performed; that is:

$$\mathsf{val} = \mathsf{s}(0)$$

In both GrB_BLOCKING mode GrB_NONBLOCKING mode if the method exits with return value GrB_SUCCESS, the new contents of val are as defined above.

4.2.3 Vector methods

4.2.3.1 Vector_new: Construct new vector

Creates a new vector with specified domain and size.

C Syntax

```
GrB_Info GrB_Vector_new(GrB_Vector *v,
                        GrB_Type    d,
                        GrB_Index   nsize);
```

Parameters

- v (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.
- d (IN) The type corresponding to the domain of the vector being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.
- nsize (IN) The size of the vector being created.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector v is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).
- GrB_NULL_POINTER The v pointer is NULL.
- GrB_INVALID_VALUE nsize is zero or outside the range of the type GrB_Index.

Description

Creates a new vector $\mathbf{v}$ of domain $\mathbf{D}(\mathsf{d})$, size nsize, and empty $\mathbf{L}(\mathbf{v})$. The method returns a handle to the new vector in v.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.3.2 Vector_dup: Construct a copy of a GraphBLAS vector

Creates a new vector with the same domain, size, and contents as another vector.

C Syntax

```
GrB_Info GrB_Vector_dup(GrB_Vector       *w,
                        const GrB_Vector  u);
```

Parameters

- w (INOUT) On successful return, contains a handle to the newly created GraphBLAS vector.
- u (IN) The GraphBLAS vector to be duplicated.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	The GraphBLAS vector, u, has not been initialized by a call to Vector_new or Vector_dup.
GrB_NULL_POINTER	The w pointer is NULL.

Description

Creates a new vector **w** of domain **D**(u), size **size**(u), and contents **L**(u). The method returns a handle to the new vector in w.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.3.3 Vector_resize: Resize a vector

Changes the size of an existing vector.

C Syntax

```
GrB_Info GrB_Vector_resize(GrB_Vector  w,
                           GrB_Index   nsize);
```

Parameters

- w (INOUT) An existing Vector object that is being resized.
- nsize (IN) The new size of the vector. It can be smaller or larger than the current size.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_NULL_POINTER	The w pointer is NULL.
GrB_INVALID_VALUE	nsize is zero or outside the range of the type GrB_Index.

Description

Changes the size of w to nsize. The domain $\mathbf{D}(\mathsf{w})$ of vector w remains the same. The contents $\mathbf{L}(\mathsf{w})$ are modified as described below.

Let $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), N, \mathbf{L}(\mathsf{w}) \rangle$ when the method is called. When the method returns, $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathsf{nsize}, \mathbf{L}'(\mathsf{w}) \rangle$ where $\mathbf{L}'(\mathsf{w}) = \{(i, w_i) : (i, w_i) \in \mathbf{L}(\mathsf{w}) \wedge (i < \mathsf{nsize})\}$. That is, all elements of w with index greater than or equal to the new vector size (nsize) are dropped.

4.2.3.4 Vector_clear: Clear a vector

Removes all the elements (tuples) from a vector.

C Syntax

```
GrB_Info GrB_Vector_clear(GrB_Vector v);
```

Parameters

v (INOUT) An existing GraphBLAS vector to clear.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector v is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.

Description

Removes all elements (tuples) from an existing vector. After the call to GrB_Vector_clear(v), $\mathbf{L}(\mathbf{v}) = \emptyset$. The size of the vector does not change.

4.2.3.5 Vector_size: Size of a vector

Retrieve the size of a vector.

C Syntax

```
GrB_Info GrB_Vector_size(GrB_Index        *nsize,
                         const GrB_Vector  v);
```

Parameters

- nsize (OUT) On successful return, is set to the size of the vector.
- v (IN) An existing GraphBLAS vector being queried.

Return Values

- GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nsize has been set.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.
- GrB_NULL_POINTER nsize pointer is NULL.

Description

Return **size**(v) in nsize.

4.2.3.6 Vector_nvals: Number of stored elements in a vector

Retrieve the number of stored elements (tuples) in a vector.

C Syntax

```
GrB_Info GrB_Vector_nvals(GrB_Index        *nvals,
                          const GrB_Vector  v);
```

Parameters

- nvals (OUT) On successful return, this is set to the number of stored elements (tuples) in the vector.
- v (IN) An existing GraphBLAS vector being queried.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.

GrB_NULL_POINTER The nvals pointer is NULL.

Description

Return **nvals**(v) in nvals. This is the number of stored elements in vector v, which is the size of $\mathbf{L}(\mathbf{v})$ (see Section 3.5.2).

4.2.3.7 Vector_build: Store elements from tuples into a vector

C Syntax

```
GrB_Info GrB_Vector_build(GrB_Vector             w,
                          const GrB_Index       *indices,
                          const <type>          *values,
                          GrB_Index              n,
                          const GrB_BinaryOp     dup);
```

Parameters

w (INOUT) An existing Vector object to store the result.

indices (IN) Pointer to an array of indices.

values (IN) Pointer to an array of scalars of a type that is compatible with the domain of vector w.

n (IN) The number of entries contained in each array (the same for indices and values).

dup (IN) An associative and commutative binary operator to apply when duplicate values for the same location are present in the input arrays. All three domains of dup must be the same; hence $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$. If dup is GrB_NULL, then duplicate locations will result in an error.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT Either w has not been initialized by a call to by GrB_Vector_new or by GrB_Vector_dup, or dup has not been initialized by a call to by GrB_BinaryOp_new.

GrB_NULL_POINTER indices or values pointer is NULL.

GrB_INDEX_OUT_OF_BOUNDS A value in indices is outside the allowed range for w.

GrB_DOMAIN_MISMATCH Either the domains of the GraphBLAS binary operator dup are not all the same, or the domains of values and w are incompatible with each other or D_{dup}.

GrB_OUTPUT_NOT_EMPTY Output vector w already contains valid tuples (elements). In other words, GrB_Vector_nvals(C) returns a positive value.

GrB_INVALID_VALUE indices contains a duplicate location and dup is GrB_NULL.

Description

If dup is not GrB_NULL, an internal vector $\widetilde{\mathbf{w}} = \langle D_{dup}, \mathbf{size}(\mathsf{w}), \emptyset \rangle$ is created, which only differs from w in its domain; otherwise, $\widetilde{\mathbf{w}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \emptyset \rangle$.

Each tuple $\{\mathsf{indices[k]}, \mathsf{values[k]}\}$, where $0 \leq k < \mathsf{n}$, is a contribution to the output in the form of

$$\widetilde{\mathbf{w}}(\mathsf{indices[k]}) = \begin{cases} (D_{dup})\,\mathsf{values[k]} & \text{if } \mathsf{dup} \neq \mathsf{GrB\_NULL} \\ (\mathbf{D}(\mathsf{w}))\,\mathsf{values[k]} & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB_NULL, dup is used to reduce the values before assignment into $\widetilde{\mathbf{w}}$ as follows:

$$\widetilde{\mathbf{w}}_i = \bigoplus_{k:\,\mathsf{indices[k]}=i} (D_{dup})\,\mathsf{values[k]},$$

where $\oplus$ is the dup binary operator. Finally, the resulting $\widetilde{\mathbf{w}}$ is copied into w via typecasting its values to $\mathbf{D}(\mathsf{w})$ if necessary. If $\oplus$ is not associative or not commutative, the result is undefined.

The nonopaque input arrays, indices and values, must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB_Vector_nvals(w) should evaluate to zero prior to calling this function.

After GrB_Vector_build returns, it is safe for a programmer to modify or delete the arrays indices or values.

4.2.3.8 Vector_setElement: Set a single element in a vector

Set one element of a vector to a given value.

C Syntax

```
// scalar value
GrB_Info GrB_Vector_setElement(GrB_Vector        w,
                               <type>            val,
                               GrB_Index         index);

// GraphBLAS scalar
GrB_Info GrB_Vector_setElement(GrB_Vector        w,
                               const GrB_Scalar  s,
                               GrB_Index         index);
```

Parameters

- w (INOUT) An existing GraphBLAS vector for which an element is to be assigned.
- val or s (IN) Scalar assign. Its domain (type) must be compatible with the domain of w.
- index (IN) The location of the element to be assigned.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, w, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.

GrB_INVALID_INDEX index specifies a location that is outside the dimensions of w.

GrB_DOMAIN_MISMATCH The domains of the vector and the scalar are incompatible.

Description

First, the scalar and output vector are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{w})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Vector_setElement ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

$$0 \ \leq \ \mathsf{index} \ < \ \mathbf{size}(\mathsf{w})$$

If this condition is violated, execution of GrB_Vector_setElement ends and the invalid index error listed above is returned.

We are now ready to carry out the assignment; that is:

$$\mathsf{w}(\mathsf{index}) = \begin{cases} \mathbf{L}(\mathsf{s}), & \text{GraphBLAS scalar.} \\ \mathsf{val}, & \text{otherwise.} \end{cases}$$

In the case of a transparent scalar or if $\mathbf{L}(\mathsf{s})$ is not empty, then a value will be stored at the specified location in w, overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar, if $\mathbf{L}(\mathsf{s})$ is empty, then any value stored at the specified location in w will be removed.

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.3.9 Vector_removeElement: Remove an element from a vector

Remove (annihilate) one stored element from a vector.

C Syntax

```
        GrB_Info GrB_Vector_removeElement(GrB_Vector   w,
                                          GrB_Index    index);
```

Parameters

- w (INOUT) An existing GraphBLAS vector from which an element is to be removed.
- index (IN) The location of the element to be removed.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output vector w is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, w, has not been initialized by a call to Vector_new or Vector_dup.
- GrB_INVALID_INDEX index specifies a location that is outside the dimensions of w.

Description

First, the index parameter is checked for a valid value where the following condition must hold:

$$0 \leq \mathsf{index} < \mathbf{size}(\mathsf{w})$$

If this condition is violated, execution of GrB_Vector_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by index. If a value does not exist at the specified location in w, no error is reported and the operation has no effect on the state of w. In either case, the following will be true on return from the method: index $\notin$ **ind**(w).

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.3.10 Vector_extractElement: Extract a single element from a vector.

Extract one element of a vector into a scalar.

C Syntax

```
// scalar value
GrB_Info GrB_Vector_extractElement(<type>           *val,
                                   const GrB_Vector  u,
                                   GrB_Index         index);

// GraphBLAS scalar
GrB_Info GrB_Vector_extractElement(GrB_Scalar        s,
                                   const GrB_Vector  u,
                                   GrB_Index         index);
```

Parameters

- val or s (INOUT) An existing scalar of whose domain is compatible with the domain of vector u. On successful return, this scalar holds the result of the extract. Any previous value stored in val or s is overwritten.
- u (IN) The GraphBLAS vector from which an element is extracted.
- index (IN) The location in u to extract.

Return Values

- GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val or s, has been computed and is ready to be used in the next method of the sequence.

GrB_NO_VALUE When using the transparent scalar, val, this is returned when there is no stored value at specified location.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, u, or scalar, s, has not been initialized by a call to a corresponding constructor.

GrB_NULL_POINTER val pointer is NULL.

GrB_INVALID_INDEX index specifies a location that is outside the dimensions of w.

GrB_DOMAIN_MISMATCH The domains of the vector and scalar are incompatible.

Description

First, the scalar and input vector are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{u})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Vector_extractElement ends and the domain mismatch error listed above is returned.

Then, the index parameter is checked for a valid value where the following condition must hold:

$$0 \leq \mathsf{index} < \mathbf{size}(\mathsf{u})$$

If this condition is violated, execution of GrB_Vector_extractElement ends and the invalid index error listed above is returned.

We are now ready to carry out the extract into the output scalar; that is:

$$\left.\begin{array}{r}\mathbf{L}(\mathsf{s})\\ \mathsf{val}\end{array}\right\} = \mathsf{u}(\mathsf{index})$$

If $\mathsf{index} \in \mathbf{ind}(\mathsf{u})$, then the corresponding value from u is copied into s or val with casting as necessary. If $\mathsf{index} \notin \mathbf{ind}(\mathsf{u})$, then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB_SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB_NO_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB_BLOCKING mode GrB_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB_SUCCESS or GrB_NO_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB_BLOCKING mode. In GrB_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.3.11 Vector_extractTuples: Extract tuples from a vector

Extract the contents of a GraphBLAS vector into non-opaque data structures.

C Syntax

```
GrB_Info GrB_Vector_extractTuples(GrB_Index            *indices,
                                  <type>               *values,
                                  GrB_Index            *n,
                                  const GrB_Vector      v);
```

- **indices** (OUT) Pointer to an array of indices that is large enough to hold all of the stored values' indices.
- **values** (OUT) Pointer to an array of scalars of a type that is large enough to hold all of the stored values whose type is compatible with $\mathbf{D}(\mathbf{v})$.
- **n** (INOUT) Pointer to a value indicating (on input) the number of elements the values and indices arrays can hold. Upon return, it will contain the number of values written to the arrays.
- **v** (IN) An existing GraphBLAS vector.

Return Values

- **GrB_SUCCESS** In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indices and values, have been computed.
- **GrB_PANIC** Unknown internal error.
- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_INSUFFICIENT_SPACE Not enough space in indices and values (as indicated by the n parameter) to hold all of the tuples that will be extacted.

GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.

GrB_NULL_POINTER indices, values, or n pointer is NULL.

GrB_DOMAIN_MISMATCH The domains of the v vector or values array are incompatible with one another.

Description

This method will extract all the tuples from the GraphBLAS vector v. The values associated with those tuples are placed in the values array and the indices are placed in the indices array. Both indices and values must be pre-allocated by the user to have enough space to hold at least GrB_Vector_nvals(v) elements before calling this function.

Upon return of this function, n will be set to the number of values (and indices) copied. Also, the entries of indices are unique, but not necessarily sorted. Each tuple (i, v_i) in v is unzipped and copied into a distinct kth location in output vectors:

$$\{\mathsf{indices[k]}, \mathsf{values[k]}\} \leftarrow (i, v_i),$$

where $0 \leq k <$ GrB_Vector_nvals(v). No gaps in output vectors are allowed; that is, if indices[k] and values[k] exist upon return, so does indices[j] and values[j] for all j such that $0 \leq j < k$.

Note that if the value in n on input is less than the number of values contained in the vector v, then a GrB_INSUFFICIENT_SPACE error is returned because it is undefined which subset of values would be extracted otherwise.

In both GrB_BLOCKING mode GrB_NONBLOCKING mode if the method exits with return value GrB_SUCCESS, the new contents of the arrays indices and values are as defined above.

4.2.4 Matrix methods

4.2.4.1 Matrix_new: Construct new matrix

Creates a new matrix with specified domain and dimensions.

C Syntax

```
        GrB_Info GrB_Matrix_new(GrB_Matrix *A,
                                GrB_Type    d,
```

```
                                  GrB_Index   nrows,
                                  GrB_Index   ncols);
```

Parameters

- A (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.
- d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.
- nrows (IN) The number of rows of the matrix being created.
- ncols (IN) The number of columns of the matrix being created.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).
- GrB_NULL_POINTER The A pointer is NULL.
- GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index.

Description

Creates a new matrix $\mathbf{A}$ of domain $\mathbf{D}(\mathsf{d})$, size $\mathsf{nrows} \times \mathsf{ncols}$, and empty $\mathbf{L}(\mathbf{A})$. The method returns a handle to the new matrix in A.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.4.2 Matrix_dup: Construct a copy of a GraphBLAS matrix

Creates a new matrix with the same domain, dimensions, and contents as another matrix.

C Syntax

```
GrB_Info GrB_Matrix_dup(GrB_Matrix       *C,
                        const GrB_Matrix  A);
```

Parameters

- C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix.
- A (IN) The GraphBLAS matrix to be duplicated.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.
- GrB_NULL_POINTER The C pointer is NULL.

Description

Creates a new matrix $\mathbf{C}$ of domain $\mathbf{D}(\mathsf{A})$, size $\mathbf{nrows}(\mathsf{A}) \times \mathbf{ncols}(\mathsf{A})$, and contents $\mathbf{L}(\mathsf{A})$. It returns a handle to it in C.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.4.3 Matrix_diag: Construct a diagonal GraphBLAS matrix

Creates a new matrix with the same domain and contents as a GrB_Vector, and square dimensions appropriate for placing the contents of the vector along the specified diagonal of the matrix.

C Syntax

```
        GrB_Info GrB_Matrix_diag(GrB_Matrix       *C,
                                 const GrB_Vector  v,
                                 int64_t           k);
```

Parameters

- C (INOUT) On successful return, contains a handle to the newly created GraphBLAS matrix. The matrix is square with each dimension equal to $\mathbf{size}(\mathsf{v}) + |k|$.
- v (IN) The GraphBLAS vector whose contents will be copied to the diagonal of the matrix.
- k (IN) The diagonal to which the vector is assigned. $\mathsf{k} = 0$ represents the main diagonal, $\mathsf{k} > 0$ is above the main diagonal, and $\mathsf{k} < 0$ is below.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for the operation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS vector, v, has not been initialized by a call to Vector_new or Vector_dup.
- GrB_NULL_POINTER The C pointer is NULL.

Description

Creates a new matrix $\mathbf{C}$ of domain $\mathbf{D}(\mathsf{v})$, size $(\mathbf{size}(\mathsf{v}) + |k|) \times (\mathbf{size}(\mathsf{v}) + |k|)$, and contents

$$\begin{aligned} \mathbf{L}(\mathsf{C}) &= \{(i, i+k, v_i) \ : \ (i, v_i) \in \mathbf{L}(\mathsf{v})\} \text{ if } k \geq 0 \text{ or} \\ \mathbf{L}(\mathsf{C}) &= \{(i-k, i, v_i) \ : \ (i, v_i) \in \mathbf{L}(\mathsf{v})\} \text{ if } k \geq 0. \end{aligned}$$

It returns a handle to it in C. It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.4.4 Matrix_resize: Resize a matrix

Changes the dimensions of an existing matrix.

C Syntax

```
GrB_Info GrB_Matrix_resize(GrB_Matrix  C,
                           GrB_Index   nrows,
                           GrB_Index   ncols);
```

Parameters

- C (INOUT) An existing Matrix object that is being resized.
- nrows (IN) The new number of rows of the matrix. It can be smaller or larger than the current number of rows.
- ncols (IN) The new number of columns of the matrix. It can be smaller or larger than the current number of columns.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_NULL_POINTER The C pointer is NULL.

GrB_INVALID_VALUE nrows or ncols is zero or outside the range of the type GrB_Index.

Description

Changes the number of rows and columsn of C to nrows and ncols, respectively. The domain $\mathbf{D}(\mathsf{C})$ of matrix C remains the same. The contents $\mathbf{L}(\mathsf{C})$ are modified as described below.

Let $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), M, N, \mathbf{L}(\mathsf{C})\rangle$ when the method is called. When the method returns C is modified to $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathsf{nrows}, \mathsf{ncols}, \mathbf{L}'(\mathsf{C})\rangle$ where $\mathbf{L}'(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j, C_{ij}) \in \mathbf{L}(\mathsf{C}) \wedge (i < \mathsf{nrows}) \wedge (j < \mathsf{ncols})\}$. That is, all elements of C with row index greater than or equal to nrows or column index greater than or equal to ncols are dropped.

4.2.4.5 Matrix_clear: Clear a matrix

Removes all elements (tuples) from a matrix.

C Syntax

```
GrB_Info GrB_Matrix_clear(GrB_Matrix A);
```

Parameters

A (IN) An exising GraphBLAS matrix to clear.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

Description

Removes all elements (tuples) from an existing matrix. After the call to GrB_Matrix_clear(A), $\mathbf{L}(\mathbf{A}) = \emptyset$. The dimensions of the matrix do not change.

4.2.4.6 Matrix_nrows: Number of rows in a matrix

Retrieve the number of rows in a matrix.

C Syntax

```
        GrB_Info GrB_Matrix_nrows(GrB_Index        *nrows,
                                  const GrB_Matrix  A);
```

Parameters

- nrows (OUT) On successful return, contains the number of rows in the matrix.
- A (IN) An existing GraphBLAS matrix being queried.

Return Values

GrB_SUCCESS	In blocking or non-blocking mode, the operation completed successfully and the value of nrows has been set.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_UNINITIALIZED_OBJECT	The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.
GrB_NULL_POINTER	nrows pointer is NULL.

Description

Return **nrows**(A) in nrows (the number of rows).

4.2.4.7 Matrix_ncols: Number of columns in a matrix

Retrieve the number of columns in a matrix.

C Syntax

```
GrB_Info GrB_Matrix_ncols(GrB_Index        *ncols,
                          const GrB_Matrix  A);
```

Parameters

- ncols (OUT) On successful return, contains the number of columns in the matrix.
- A (IN) An existing GraphBLAS matrix being queried.

Return Values

- GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of ncols has been set.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.
- GrB_NULL_POINTER ncols pointer is NULL.

Description

Return **ncols**(A) in ncols (the number of columns).

4.2.4.8 Matrix_nvals: Number of stored elements in a matrix

Retrieve the number of stored elements (tuples) in a matrix.

C Syntax

```
GrB_Info GrB_Matrix_nvals(GrB_Index        *nvals,
                          const GrB_Matrix  A);
```

Parameters

nvals (OUT) On successful return, contains the number of stored elements (tuples) in the matrix.

A (IN) An existing GraphBLAS matrix being queried.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of nvals has been set.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER The nvals pointer is NULL.

Description

Return **nvals**(A) in nvals. This is the number of tuples stored in matrix A, which is the size of $\mathbf{L}(\mathbf{A})$ (see Section 3.5.3).

4.2.4.9 Matrix_build: Store elements from tuples into a matrix

C Syntax

```
GrB_Info GrB_Matrix_build(GrB_Matrix              C,
                          const GrB_Index       *row_indices,
                          const GrB_Index       *col_indices,
                          const <type>          *values,
                          GrB_Index              n,
                          const GrB_BinaryOp     dup);
```

Parameters

C (INOUT) An existing Matrix object to store the result.

row_indices (IN) Pointer to an array of row indices.

col_indices (IN) Pointer to an array of column indices.

values (IN) Pointer to an array of scalars of a type that is compatible with the domain of matrix, C.

n (IN) The number of entries contained in each array (the same for row_indices, col_indices, and values).

dup (IN) An associative and commutative binary operator to apply when duplicate values for the same location are present in the input arrays. All three domains of dup must be the same; hence $dup = \langle D_{dup}, D_{dup}, D_{dup}, \oplus \rangle$. If dup is GrB_NULL, then duplicate locations will result in an error.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT Either C has not been initialized by a call to any matrix constructor, or dup has not been initialized by a call to by GrB_BinaryOp_new.

GrB_NULL_POINTER row_indices, col_indices or values pointer is NULL.

GrB_INDEX_OUT_OF_BOUNDS A value in row_indices or col_indices is outside the allowed range for C.

GrB_DOMAIN_MISMATCH Either the domains of the GraphBLAS binary operator dup are not all the same, or the domains of values and C are incompatible with each other or D_{dup}.

GrB_OUTPUT_NOT_EMPTY Output matrix C already contains valid tuples (elements). In other words, GrB_Matrix_nvals(C) returns a positive value.

GrB_INVALID_VALUE indices contains a duplicate location and dup is GrB_NULL.

Description

If dup is not GrB_NULL, an internal matrix $\widetilde{\mathbf{C}} = \langle D_{dup}, \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$ is created, which only differs from C in its domain; otherwise, $\widetilde{\mathbf{C}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \emptyset \rangle$.

Each tuple $\{\mathsf{row\_indices[k]}, \mathsf{col\_indices[k]}, \mathsf{values[k]}\}$, where $0 \leq k < \mathsf{n}$, is a contribution to the output in the form of

$$\widetilde{\mathbf{C}}(\mathsf{row\_indices[k]}, \mathsf{col\_indices[k]}) = \begin{cases} (D_{dup})\,\mathsf{values[k]} & \text{if } \mathsf{dup} \neq \mathsf{GrB\_NULL} \\ (\mathbf{D}(\mathsf{C}))\,\mathsf{values[k]} & \text{otherwise.} \end{cases}$$

If multiple values for the same location are present in the input arrays and dup is not GrB_NULL, dup is used to reduce the values before assignment into $\widetilde{\mathbf{C}}$ as follows:

$$\widetilde{\mathbf{C}}_{ij} = \bigoplus_{k:\, \mathsf{row\_indices[k]}=i \,\wedge\, \mathsf{col\_indices[k]}=j} (D_{dup})\,\mathsf{values[k]},$$

where $\oplus$ is the dup binary operator. Finally, the resulting $\widetilde{\mathbf{C}}$ is copied into C via typecasting its values to $\mathbf{D}(\mathsf{C})$ if necessary. If $\oplus$ is not associative or not commutative, the result is undefined.

The nonopaque input arrays row_indices, col_indices, and values must be at least as large as n.

It is an error to call this function on an output object with existing elements. In other words, GrB_Matrix_nvals(C) should evaluate to zero prior to calling this function.

After GrB_Matrix_build returns, it is safe for a programmer to modify or delete the arrays row_indices, col_indices, or values.

4.2.4.10 Matrix_setElement: Set a single element in matrix

Set one element of a matrix to a given value.

C Syntax

```
// scalar value
GrB_Info GrB_Matrix_setElement(GrB_Matrix         C,
                               <type>             val,
                               GrB_Index          row_index,
                               GrB_Index          col_index);

// GraphBLAS scalar
GrB_Info GrB_Matrix_setElement(GrB_Matrix         C,
                               const GrB_Scalar   s,
                               GrB_Index          row_index,
                               GrB_Index          col_index);
```

Parameters

- C (INOUT) An existing GraphBLAS matrix for which an element is to be assigned.
- val or s (IN) Scalar to assign. Its domain (type) must be compatible with the domain of C.
- row_index (IN) Row index of element to be assigned
- col_index (IN) Column index of element to be assigned

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, or GraphBLAS scalar, s, has not been initialized by a call to a respective constructor.
- GrB_INVALID_INDEX row_index or col_index is outside the allowable range (i.e., not less than **nrows**(C) or **ncols**(C), respectively).
- GrB_DOMAIN_MISMATCH The domains of the matrix and the scalar are incompatible.

Description

First, the scalar and output matrix are tested for domain compatibility as follows: **D**(val) or **D**(s) must be compatible with **D**(C). Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Matrix_setElement ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

$$\begin{aligned} 0 \ \leq \ \mathsf{row\_index} \ &< \ \mathbf{nrows}(\mathsf{C}), \\ 0 \ \leq \ \mathsf{col\_index} \ &< \ \mathbf{ncols}(\mathsf{C}) \end{aligned}$$

If either of these conditions is violated, execution of GrB_Matrix_setElement ends and the invalid index error listed above is returned.

We are now ready to carry out the assignment; that is:

$$\mathsf{C}(\mathsf{row\_index}, \mathsf{col\_index}) = \begin{cases} \mathbf{L}(\mathsf{s}), & \text{GraphBLAS scalar.} \\ \mathsf{val}, & \text{otherwise.} \end{cases}$$

In the case of a transparent scalar or if $\mathbf{L}(\mathsf{s})$ is not empty, then a value will be stored at the specified location in C, overwriting any value that may have been stored there before. In the case of a GraphBLAS scalar and if $\mathbf{L}(\mathsf{s})$ is empty, then any value stored at the specified location in C will be removed.

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.4.11 Matrix_removeElement: Remove an element from a matrix

Remove (annihilate) one stored element from a matrix.

C Syntax

```
GrB_Info GrB_Matrix_removeElement(GrB_Matrix   C,
                                  GrB_Index    row_index,
                                  GrB_Index    col_index);
```

Parameters

- **C** (INOUT) An existing GraphBLAS matrix from which an element is to be removed.
- **row_index** (IN) Row index of element to be removed
- **col_index** (IN) Column index of element to be removed

Return Values

- **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on index/dimensions and domains for the input arguments passed successfully. Either way, the output matrix C is ready to be used in the next method of the sequence.
- **GrB_PANIC** Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, C, has not been initialized by a call to any matrix constructor.

GrB_INVALID_INDEX row_index or col_index is outside the allowable range (i.e., not less than **nrows**(C) or **ncols**(C), respectively).

Description

First, both index parameters are checked for valid values where following conditions must hold:

$$0 \leq \text{row\_index} < \textbf{nrows}(\mathsf{C}),$$
$$0 \leq \text{col\_index} < \textbf{ncols}(\mathsf{C})$$

If either of these conditions is violated, execution of GrB_Matrix_removeElement ends and the invalid index error listed above is returned.

We are now ready to carry out the removal of a value that may be stored at the location specified by (row_index, col_index). If a value does not exist at the specified location in C, no error is reported and the operation has no effect on the state of C. In either case, the following will be true on return from this method: $(\text{row\_index}, \text{col\_index}) \notin \textbf{ind}(\mathsf{C})$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new contents of C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector C is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.4.12 Matrix_extractElement: Extract a single element from a matrix

Extract one element of a matrix into a scalar.

C Syntax

```
// scalar value
GrB_Info GrB_Matrix_extractElement(<type>            *val,
                                   const GrB_Matrix  A,
                                   GrB_Index         row_index,
                                   GrB_Index         col_index);

// GraphBLAS scalar
```

```
GrB_Info GrB_Matrix_extractElement(GrB_Scalar        s,
                                   const GrB_Matrix  A,
                                   GrB_Index         row_index,
                                   GrB_Index         col_index);
```

Parameters

- **val or s** (INOUT) An existing scalar whose domain is compatible with the domain of matrix A. On successful return, this scalar holds the result of the extract. Any previous value stored in val or s is overwritten.
- **A** (IN) The GraphBLAS matrix from which an element is extracted.
- **row_index** (IN) The row index of location in A to extract.
- **col_index** (IN) The column index of location in A to extract.

Return Values

- **GrB_SUCCESS** In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully, and the output scalar, val or s, has been computed and is ready to be used in the next method of the sequence.
- **GrB_NO_VALUE** When using the transparent scalar, val, this is returned when there is no stored value at specified location.
- **GrB_PANIC** Unknown internal error.
- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- **GrB_OUT_OF_MEMORY** Not enough memory available for operation.
- **GrB_UNINITIALIZED_OBJECT** The GraphBLAS matrix, A, or scalar, s, has not been initialized by a call to a corresponding constructor.
- **GrB_NULL_POINTER** val pointer is NULL.
- **GrB_INVALID_INDEX** row_index or col_index is outside the allowable range (i.e. less than zero or greater than or equal to **nrows**(A) or **ncols**(A), respectively).
- **GrB_DOMAIN_MISMATCH** The domains of the matrix and scalar are incompatible.

Description

First, the scalar and input matrix are tested for domain compatibility as follows: $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{A})$. Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_Matrix_extractElement ends and the domain mismatch error listed above is returned.

Then, both index parameters are checked for valid values where following conditions must hold:

$$0 \le \mathsf{row\_index} < \mathbf{nrows}(\mathsf{A}),$$
$$0 \le \mathsf{col\_index} < \mathbf{ncols}(\mathsf{A})$$

If either condition is violated, execution of GrB_Matrix_extractElement ends and the invalid index error listed above is returned.

We are now ready to carry out the extract into the output scalar; that is,

$$\left.\begin{array}{r}\mathbf{L}(\mathsf{s})\\ \mathsf{val}\end{array}\right\} = \mathsf{A}(\mathsf{row\_index}, \mathsf{col\_index})$$

If $(\mathsf{row\_index}, \mathsf{col\_index}) \in \mathbf{ind}(\mathsf{A})$, then the corresponding value from A is copied into s or val with casting as necessary. If $(\mathsf{row\_index}, \mathsf{col\_index}) \notin \mathbf{ind}(\mathsf{A})$, then one of the follow occurs depending on output scalar type:

- The GraphBLAS scalar, s, is cleared and GrB_SUCCESS is returned.
- The non-opaque scalar, val, is unchanged, and GrB_NO_VALUE is returned.

When using the non-opaque scalar variant (val) in both GrB_BLOCKING mode GrB_NONBLOCKING mode, the new contents of val are as defined above if the method exits with return value GrB_SUCCESS or GrB_NO_VALUE.

When using the GraphBLAS scalar variant (s) with a GrB_SUCCESS return value, the method exits and the new contents of s is as defined above and fully computed in GrB_BLOCKING mode. In GrB_NONBLOCKING mode, the new contents of s is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.2.4.13 Matrix_extractTuples: Extract tuples from a matrix

Extract the contents of a GraphBLAS matrix into non-opaque data structures.

C Syntax

```
    GrB_Info GrB_Matrix_extractTuples(GrB_Index            *row_indices,
                                      GrB_Index            *col_indices,
```

```
                                            <type>              *values,
                                            GrB_Index           *n,
                                            const GrB_Matrix     A);
```

Parameters

- **row_indices** (OUT) Pointer to an array of row indices that is large enough to hold all of the row indices.
- **col_indices** (OUT) Pointer to an array of column indices that is large enough to hold all of the column indices.
- **values** (OUT) Pointer to an array of scalars of a type that is large enough to hold all of the stored values whose type is compatible with $\mathbf{D}(\mathbf{A})$.
- **n** (INOUT) Pointer to a value indicating (in input) the number of elements the values, row_indices, and col_indices arrays can hold. Upon return, it will contain the number of values written to the arrays.
- **A** (IN) An existing GraphBLAS matrix.

Return Values

- **GrB_SUCCESS** In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indices and values, have been computed.
- **GrB_PANIC** Unknown internal error.
- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- **GrB_OUT_OF_MEMORY** Not enough memory available for operation.
- **GrB_INSUFFICIENT_SPACE** Not enough space in row_indices, col_indices, and values (as indicated by the n parameter) to hold all of the tuples that will be extacted.
- **GrB_UNINITIALIZED_OBJECT** The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.
- **GrB_NULL_POINTER** row_indices, col_indices, values or n pointer is NULL.
- **GrB_DOMAIN_MISMATCH** The domains of the A matrix and values array are incompatible with one another.

Description

This method will extract all the tuples from the GraphBLAS matrix A. The values associated with those tuples are placed in the values array, the column indices are placed in the col_indices array, and the row indices are placed in the row_indices array. These output arrays are pre-allocated by the user before calling this function such that each output array has enough space to hold at least GrB_Matrix_nvals(A) elements.

Upon return of this function, a pair of {row_indices[k], col_indices[k]} are unique for every valid k, but they are not required to be sorted in any particular order. Each tuple (i, j, A_{ij}) in A is unzipped and copied into a distinct kth location in output vectors:

$$\{\mathsf{row\_indices[k]}, \mathsf{col\_indices[k]}, \mathsf{values[k]}\} \leftarrow (i, j, A_{ij}),$$

where $0 \leq k <$ GrB_Matrix_nvals(v). No gaps in output vectors are allowed; that is, if row_indices[k], col_indices[k] and values[k] exist upon return, so does row_indices[j], col_indices[j] and values[j] for all j such that $0 \leq j < k$.

Note that if the value in n on input is less than the number of values contained in the matrix A, then a GrB_INSUFFICIENT_SPACE error is returned since it is undefined which subset of values would be extracted.

In both GrB_BLOCKING mode GrB_NONBLOCKING mode if the method exits with return value GrB_SUCCESS, the new contents of the arrays row_indices, col_indices and values are as defined above.

4.2.4.14 Matrix_exportHint: Provide a hint as to which storage format might be most efficient for exporting a matrix

C Syntax

```
GrB_Info GrB_Matrix_exportHint(GrB_Format            *hint,
                               GrB_Matrix             A);
```

Parameters

- hint (OUT) Pointer to a value of type GrB_Format.
- A (IN) A GraphBLAS matrix object.

Return Values

- GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully and the value of hint has been set.
- GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER hint is NULL.

GrB_NO_VALUE If the implementation does not have a preferred format, it may return the value GrB_NO_VALUE.

Description

Given a GraphBLAS matrix A, provide a hint as to which format might be most efficient for exporting the matrix A. GraphBLAS implementations might return the current storage format of the matrix, or the format to which it could most efficiently be exported. However, implementations are free to return any value for format defined in Section 3.5.3.1. Note that an implementation is free to refuse to provide a format hint, returning GrB_NO_VALUE.

4.2.4.15 Matrix_exportSize: Return the array sizes necessary to export a GraphBLAS matrix object

C Syntax

```
GrB_Info GrB_Matrix_exportSize(GrB_Index             *n_indptr,
                               GrB_Index             *n_indices,
                               GrB_Index             *n_values,
                               GrB_Format             format,
                               GrB_Matrix             A);
```

Parameters

n_indptr (OUT) Pointer to a value of type GrB_Index.

n_indices (OUT) Pointer to a value of type GrB_Index.

n_values (OUT) Pointer to a value of type GrB_Index.

format (IN) a value indicating the format in which the matrix will be exported, as defined in Section 3.5.3.1.

A (IN) A GraphBLAS matrix object.

Return Values

GrB_SUCCESS	In blocking mode or non-blocking mode, the operation completed successfully. This indicates that the API checks for the input arguments passed successfully, and the number of elements necessary for the export buffers have been written to n_indptr, n_indices, and n_values, respectively.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	The GraphBLAS Matrix, A, has not been initialized by a call to any matrix constructor.
GrB_NULL_POINTER	n_indptr, n_indices, or n_values is NULL.

Description

Given a matrix **A**, returns the required capacities of arrays values, indptr, and indices necessary to export the matrix in the format specified by format. The output values n_values, n_indptr, and indices will contain the corresponding sizes of the arrays (in number of elements) that must be allocated to hold the exported matrix. The argument format can be chosen arbitrarily by the user as one of the values defined in Section 3.5.3.1.

4.2.4.16 Matrix_export: Export a GraphBLAS matrix to a pre-defined format

C Syntax

```
GrB_Info GrB_Matrix_export(GrB_Index        *indptr,
                           GrB_Index        *indices,
                           <type>           *values,
                           GrB_Index        *n_indptr,
                           GrB_Index        *n_indices,
                           GrB_Index        *n_values,
                           GrB_Format        format,
                           GrB_Matrix        A);
```

Parameters

- **indptr** (INOUT) Pointer to an array that will hold row or column offsets, or row indices, depending on the value of format. It must be large enough to hold at least n_indptr elements of type GrB_Index, where n_indices was returned from GrB_Matrix_exportSize() method.

- **indices** (INOUT) Pointer to an array that will hold row or column indices of the elements in values, depending on the value of format. It must be large enough to hold at least n_indices elements of type GrB_Index, where n_indices was returned from GrB_Matrix_exportSize() method.

- **values** (INOUT) Pointer to an array that will hold stored values. The type of element must match the type of the values stored in A. It must be large enough to hold at least n_values elements of that type, where n_values was returned from GrB_Matrix_exportSize.

- **n_indptr** (INOUT) Pointer to a value indicating (on input) the number of elements the indptr array can hold. Upon return, it will contain the number of elements written to the array.

- **n_indices** (INOUT) Pointer to a value indicating (on input) the number of elements the indices array can hold. Upon return, it will contain the number of elements written to the array.

- **n_values** (INOUT) Pointer to a value indicating (on input) the number of elements the values array can hold. Upon return, it will contain the number of elements written to the array.

- **format** (IN) a value indicating the format in which the matrix will be exported, as defined in Section 3.5.3.1.

- **A** (IN) A GraphBLAS matrix object.

Return Values

- **GrB_SUCCESS** In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output arrays, indptr, indices and values, have been computed.

- **GrB_PANIC** Unknown internal error.

- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

- **GrB_OUT_OF_MEMORY** Not enough memory available for operation.

GrB_INSUFFICIENT_SPACE Not enough space in indptr, indices, and/or values (as indicated by the corresponding n_* parameter) to hold all of the corresponding elements that will be extacted.

GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.

GrB_NULL_POINTER indptr, indices, values n_indptr, n_indices, n_values pointer is NULL.

GrB_DOMAIN_MISMATCH The domain of A does not match with the type of values.

Description

Given a matrix **A**, this method exports the contents of the matrix into one of the pre-defined GrB_Format formats from Section 3.5.3.1. The user-allocated arrays pointed to by indptr, indices, and values must be at least large enough to hold the corresponding number of elements returned by calling GrB_Matrix_exportSize. The value of format can be chosen arbitrarily, but a call to GrB_Matrix_exportHint may suggest a format that results in the most efficient export. Details of the contents of indptr, indices, and values corresponding to each supported format is given in Appendix B.

4.2.4.17 Matrix_import: Import a matrix into a GraphBLAS object

C Syntax

```
GrB_Info GrB_Matrix_import(GrB_Matrix            *A,
                           GrB_Type               d,
                           GrB_Index              nrows,
                           GrB_Index              ncols
                           const GrB_Index       *indptr,
                           const GrB_Index       *indices,
                           const <type>          *values,
                           GrB_Index              n_indptr,
                           GrB_Index              n_indices,
                           GrB_Index              n_values,
                           GrB_Format             format);
```

Parameters

A (INOUT) On a successful return, contains a handle to the newly created GraphBLAS matrix.

d (IN) The type corresponding to the domain of the matrix being created. Can be one of the predefined GraphBLAS types in Table 3.2, or an existing user-defined GraphBLAS type.

nrows	(IN) Integer value holding the number of rows in the matrix.
ncols	(IN) Integer value holding the number of columns in the matrix.
indptr	(IN) Pointer to an array of row or column offsets, or row indices, depending on the value of format.
indices	(IN) Pointer to an array row or column indices of the elements in values, depending on the value of format.
values	(IN) Pointer to an array of values. Type must match the type of d.
n_indptr	(IN) Integer value holding the number of elements in the array pointed to by indptr.
n_indices	(IN) Integer value holding the number of elements in the array pointed to by indices.
n_values	(IN) Integer value holding the number of elements in the array pointed to by values.
format	(IN) a value indicating the format of the matrix being imported, as defined in Section 3.5.3.1.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully and the input arrays have been consumed. Either way, output matrix A is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).
GrB_NULL_POINTER	A, indptr, indices or values pointer is NULL.
GrB_INDEX_OUT_OF_BOUNDS	A value in indptr or indices is outside the allowed range for indices in A and or the size of values, n_values, depending on the value of format.
GrB_INVALID_VALUE	nrows or ncols is zero or outside the range of the type GrB_Index.
GrB_DOMAIN_MISMATCH	The domain given in parameter d does not match the element type of values.

Description

Creates a new matrix $\mathbf{A}$ of domain $\mathbf{D}(\mathsf{d})$ and dimension nrows $\times$ ncols. The new GraphBLAS matrix will be filled with the contents of the matrix pointed to by indptr, and indices, and values. The method returns a handle to the new matrix in A. The structure of the data being imported is defined by format, which must be equal to one of the values defined in Section 3.5.3.1. Details of the contents of indptr, indices and values for each supported format is given in Appendix B.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

4.2.4.18 Matrix_serializeSize: Compute the serialize buffer size

Compute the buffer size (in bytes) necessary to serialize a GrB_Matrix using GrB_Matrix_serialize.

C Syntax

```
GrB_Info GrB_Matrix_serializeSize(GrB_Index  *size,
                                  GrB_Matrix  A);
```

Parameters

- size (OUT) Pointer to GrB_Index value where size in bytes of serialized object will be written.
- A (IN) A GraphBLAS matrix object.

Return Values

- GrB_SUCCESS The operation completed successfully and the value pointed to by *size has been computed and is ready to use.
- GrB_PANIC Unknown internal error.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_NULL_POINTER size is NULL.

Description

Returns the size in bytes of the data buffer necessary to serialize the GraphBLAS matrix object A. Users may then allocate a buffer of size bytes to pass as a parameter to GrB_Matrix_serialize.

4.2.4.19 Matrix_serialize: Serialize a GraphBLAS matrix.

Serialize a GraphBLAS Matrix object into an opaque stream of bytes.

C Syntax

```
GrB_Info GrB_Matrix_serialize(void        *serialized_data,
                              GrB_Index   *serialized_size,
                              GrB_Matrix   A);
```

Parameters

- serialized_data (INOUT) Pointer to the preallocated buffer where the serialized matrix will be written.
- serialized_size (INOUT) On input, the size in bytes of the buffer pointed to by serialized_data. On output, the number of bytes written to serialized_data.
- A (IN) A GraphBLAS matrix object.

Return Values

- GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully. This indicates that the compatibility tests on the input argument passed successfully, and the output buffer serialized_data and serialized_size, have been computed and are ready to use.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.
- GrB_NULL_POINTER serialized_data or serialize_size is NULL.
- GrB_UNINITIALIZED_OBJECT The GraphBLAS matrix, A, has not been initialized by a call to any matrix constructor.
- GrB_INSUFFICIENT_SPACE The size of the buffer serialized_data (provided as an input serialized_size) was not large enough.

Description

Serializes a GraphBLAS matrix object to an opaque buffer. To guarantee successful execution, the size of the buffer pointed to by serialized_data, provided as an input by serialized_size, must be of at least the number of bytes returned from GrB_Matrix_serializeSize. The actual size of the serialized matrix written to serialized_data is provided upon completion as an output written to serialized_size.

The contents of the serialized buffer are implementation defined. Thus, a serialized matrix created with one library implementation is not necessarily valid for deserialization with another implementation.

4.2.4.20 Matrix_deserialize: Deserialize a GraphBLAS matrix.

Construct a new GraphBLAS matrix from a serialized object.

C Syntax

```
GrB_Info GrB_Matrix_deserialize(GrB_Matrix  *A,
                                GrB_Type     d,
                                const void  *serialized_data,
                                GrB_Index    serialized_size);
```

Parameters

- A (INOUT) On a successful return, contains a handle to the newly created GraphBLAS matrix.
- d (IN) the type of the matrix that was serialized in serialized_data.
- serialized_data (IN) a pointer to a serialized GraphBLAS matrix created with GrB_Matrix_serialize.
- serialized_size (IN) the size of the buffer pointed to by serialized_data in bytes.

Return Values

- GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the API checks for the input arguments passed successfully. Either way, output matrix A is ready to be used in the next method of the sequence.
- GrB_PANIC Unknown internal error.
- GrB_INVALID_OBJECT This is returned if serialized_data is invalid or corrupted.
- GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT	The GrB_Type object has not been initialized by a call to GrB_Type_new (needed for user-defined types).
GrB_NULL_POINTER	serialized_data or A is NULL.
GrB_DOMAIN_MISMATCH	The type given in d does not match the type of the matrix serialized in serialized_data.

Description

Creates a new matrix **A** using the serialized matrix object pointed to by serialized_data. The object pointed to by serialized_data must have been created using the method GrB_Matrix_serialize. The domain of the matrix is given as an input in d, which must match the domain of the matrix serialized in serialized_data. Note that for user-defined types, only the size of the type will be checked.

Since the format of a serialized matrix is implementation-defined, it is not guaranteed that a matrix serialized in one library implementation can be deserialized by another.

It is not an error to call this method more than once on the same output matrix; however, the handle to the previously created object will be overwritten.

4.2.5 Descriptor methods

The methods in this section create and set values in descriptors. A descriptor is an opaque GraphBLAS object the values of which are used to modify the behavior of GraphBLAS operations.

4.2.5.1 Descriptor_new: Create new descriptor

Creates a new (empty or default) descriptor.

C Syntax

```
GrB_Info GrB_Descriptor_new(GrB_Descriptor *desc);
```

Parameters

- desc (INOUT) On successful return, contains a handle to the newly created GraphBLAS descriptor.

Return Value

GrB_SUCCESS	The method completed successfully.
GrB_PANIC	unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_NULL_POINTER desc pointer is NULL.

Description

Creates a new descriptor object and returns a handle to it in desc. A newly created descriptor can be populated by calls to Descriptor_set.

It is not an error to call this method more than once on the same variable; however, the handle to the previously created object will be overwritten.

4.2.5.2 Descriptor_set: Set content of descriptor

Sets the content for a field for an existing descriptor.

C Syntax

```
GrB_Info GrB_Descriptor_set(GrB_Descriptor      desc,
                            GrB_Desc_Field      field,
                            GrB_Desc_Value      val);
```

Parameters

desc (IN) An existing GraphBLAS descriptor to be modified.

field (IN) The field being set.

val (IN) New value for the field being set.

Return Values

GrB_SUCCESS operation completed successfully.

GrB_PANIC unknown internal error.

GrB_OUT_OF_MEMORY not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT the desc parameter has not been initialized by a call to new.

GrB_INVALID_VALUE invalid value set on the field, or invalid field.

Description

For a given descriptor, the GrB_Descriptor_set method can be called for each field in the descriptor to set the value associated with that field. Valid values for the field parameter include the following:

- **GrB_OUTP** refers to the output parameter (result) of the operation.
- **GrB_MASK** refers to the mask parameter of the operation.
- **GrB_INP0** refers to the first input parameters of the operation (matrices and vectors).
- **GrB_INP1** refers to the second input parameters of the operation (matrices and vectors).

Valid values for the val parameter are:

- **GrB_STRUCTURE** Use only the structure of the stored values of the corresponding mask (GrB_MASK) parameter.
- **GrB_COMP** Use the complement of the corresponding mask (GrB_MASK) parameter. When combined with GrB_STRUCTURE, the complement of the structure of the mask is used without evaluating the values stored.
- **GrB_TRAN** Use the transpose of the corresponding matrix parameter (valid for input matrix parameters only).
- **GrB_REPLACE** When assigning the masked values to the output matrix or vector, clear the matrix first (or clear the non-masked entries). The default behavior is to leave non-masked locations unchanged. Valid for the GrB_OUTP parameter only.

Descriptor values can only be set, and once set, cannot be cleared. As, in the case of GrB_MASK, multiple values can be set and all will apply (for example, both GrB_COMP and GrB_STRUCTURE). A value for a given field may be set multiple times but will have no additional effect. Fields that have no values set result in their default behavior, as defined in Section 3.6.

4.2.6 free: Destroy an object and release its resources

Destroys a previously created GraphBLAS object and releases any resources associated with the object.

C Syntax

```
GrB_Info GrB_free(<GrB_Object> *obj);
```

Parameters

obj (INOUT) An existing GraphBLAS object to be destroyed. The object must have been created by an explicit call to a GraphBLAS constructor. It can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful completion of GrB_free, obj behaves as an uninitialized object.

Return Values

GrB_SUCCESS operation completed successfully

GrB_PANIC unknown internal error. If this return value is encountered when in nonblocking mode, the error responsible for the panic condition could be from any method involved in the computation of the input object. The GrB_error() method should be called for additional information.

Description

GraphBLAS objects consume memory and other resources managed by the GraphBLAS runtime system. A call to GrB_free frees those resources so they are available for use by other GraphBLAS objects.

The parameter passed into GrB_free is a handle referencing a GraphBLAS opaque object of a data type from table 2.1. The object must have been created by an explicit call to a GraphBLAS constructor. The behavior of a program that calls GrB_free on a pre-defined object is implementation defined.

After the GrB_free method returns, the object referenced by the input handle is destroyed and the handle has the value GrB_INVALID_HANDLE. The handle can be used in subsequent GraphBLAS methods but only after the handle has been reinitialized with a call the the appropriate _new or _dup method.

Note that unlike other GraphBLAS methods, calling GrB_free with an object with an invalid handle is legal. The system may attempt to free resources that might be associated with that object, if possible, and return normally.

When using GrB_free it is possible to create a dangling reference to an object. This would occur when a handle is assigned to a second variable of the same opaque type. This creates two handles that reference the same object. If GrB_free is called with one of the variables, the object is destroyed and the handle associated with the other variable no longer references a valid object. This is not an error condition that the implementation of the GraphBLAS API can be expected to catch, hence programmers must take care to prevent this situation from occurring.

4.2.7 wait: Return once an object is either *complete* or *materialized*

Wait until method calls in a sequence put an object into a state of *completion* or *materialization*.

C Syntax

```
GrB_Info GrB_wait(GrB_Object obj, GrB_WaitMode mode);
```

Parameters

- obj (INOUT) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type. On successful return of GrB_wait, the obj can be safely read from another thread (completion) or all computing to produce obj by all GraphBLAS operations in its sequence have finished (materialization).

- mode (IN) Set's the mode for GrB_wait for whether it is waiting for obj to be in the state of *completion* or *materialization*. Acceptable values are GrB_COMPLETE or GrB_MATERIALIZE.

Return values

- GrB_SUCCESS operation completed successfully.

- GrB_INDEX_OUT_OF_BOUNDS an index out-of-bounds execution error happened during completion of pending operations.

- GrB_OUT_OF_MEMORY and out-of-memory execution error happened during completion of pending operations.

- GrB_UNINITIALIZED_OBJECT object has not been initialized by a call to the respective *_new, or other constructor, method.

- GrB_PANIC unknown internal error.

- GrB_INVALID_VALUE method called with a GrB_WaitMode other than GrB_COMPLETE GrB_MATERIALIZE.

Description

On successful return from GrB_wait(), the input object, obj is in one of two states depending on the mode of GrB_wait:

- *complete*: obj can be used in a happens-before relation, so in a properly synchronized program it can be safely used as an IN or INOUT parameter in a GraphBLAS method call from another thread. This result occurs when the mode parameter is set to GrB_COMPLETE.

- *materialized*: obj is *complete*, but in addition, no further computing will be carried out on behalf of obj and error information is available. This result occurs when the mode parameter is set to GrB_MATERIALIZE.

Since in blocking mode OUT or INOUT parameters to any method call are materialized upon return, GrB_wait(obj,mode) has no effect when called in blocking mode.

In non-blocking mode, the status of any pending method calls, other than those associated with producing the *complete* or *materialized* state of obj, are not impacted by the call to GrB_wait(obj,mode). Methods in the sequence for obj, however, most likely would be impacted by a call to GrB_wait(obj,mode); especially in the case of the *materialized* mode for which any computing on behalf of obj must be finished prior to the return from GrB_wait(obj,mode).

4.2.8 error: Retrieve an error string

Retrieve an error-message about any errors encountered during the processing associated with an object.

C Syntax

```
GrB_Info GrB_error(const char          **error,
                   const GrB_Object      obj);
```

Parameters

error (OUT) A pointer to a null-terminated string. The contents of the string are implementation defined.

obj (IN) An existing GraphBLAS object. The object must have been created by an explicit call to a GraphBLAS constructor. Can be any of the opaque GraphBLAS objects such as matrix, vector, descriptor, semiring, monoid, binary op, unary op, or type.

Return value

GrB_SUCCESS operation completed successfully.

GrB_UNINITIALIZED_OBJECT object has not been initialized by a call to the respective *_new, or other constructor, method.

GrB_PANIC unknown internal error.

Description

This method retrieves a message related to any errors that were encountered during the last GraphBLAS method that had the opaque GraphBLAS object, obj, as an OUT or INOUT parameter. The function returns a pointer to a null-terminated string and the contents of that string are implementation-dependent. In particular, a null string (not a NULL pointer) is always a valid error string. The string that is returned is owned by obj and will be valid until the next time obj is used as an OUT or INOUT parameter or the object is freed by a call to GrB_free(obj). This is a thread-safe function. It can be safely called by multiple threads for the same object in a race-free program.

4.3 GraphBLAS operations

The GraphBLAS operations are defined in the GraphBLAS math specification and summarized in Table 4.1. In addition to methods that implement these fundamental GraphBLAS operations, we support a number of variants that have been found to be especially useful in algorithm development. A flowchart of the overall behavior of a GraphBLAS operation is shown in Figure 4.1.

Domains and Casting

A GraphBLAS operation is only valid when the domains of the GraphBLAS objects are mathematically consistent. The C programming language defines implicit casts between built-in data types. For example, floats, doubles, and ints can be freely mixed according to the rules defined for implicit casts. It is the responsibility of the user to assure that these casts are appropriate for the algorithm in question. For example, a cast to int implies truncation of a floating point type. Depending on the operation, this truncation error could lead to erroneous results. Furthermore, casting a wider type onto a narrower type can lead to overflow errors. The GraphBLAS operations do not attempt to protect a user from these sorts of errors.

When user-define types are involved, however, GraphBLAS requires strict equivalence between types and no casting is supported. If GraphBLAS detects these mismatches, it will return a domain mismatch error.

Dimensions and Transposes

GraphBLAS operations also make assumptions about the numbers of dimensions and the sizes of vectors and matrices in an operation. An operation will test these sizes and report an error if they are not *shape compatible*. For example, when multiplying two matrices, $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, the number of rows of $\mathbf{C}$ must equal the number of rows of $\mathbf{A}$, the number of columns of $\mathbf{A}$ must match the number of rows of $\mathbf{B}$, and the number of columns of $\mathbf{C}$ must match the number of columns of $\mathbf{B}$. This is the behavior expected given the mathematical definition of the operations.

For most of the GraphBLAS operations involving matrices, an optional descriptor can modify the matrix associated with an input GraphBLAS matrix object. For example, if an input matrix is an

Table 4.1: A mathematical notation for the fundamental GraphBLAS operations supported in this specification. Input matrices **A** and **B** may be optionally transposed (not shown). Use of an optional accumulate with existing values in the output object is indicated with $\odot$. Use of optional write masks and replace flags are indicated as $\mathbf{C}\langle\mathbf{M}, r\rangle$ when applied to the output matrix, **C**. The mask controls which values resulting from the operation on the right-hand side are written into the output object (complement and structure flags are not shown). The "replace" option, indicated by specifying the r flag, means that all values in the output object are removed prior to assignment. If "replace" is not specified, only the values/locations computed on the right-hand side and allowed by the mask will be written to the output ("merge" mode).

Operation Name	Mathematical Notation				
mxm	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A} \oplus . \otimes \mathbf{B}$
mxv	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$\mathbf{A} \oplus . \otimes \mathbf{u}$
vxm	$\mathbf{w}^T\langle\mathbf{m}^T, r\rangle$	$=$	$\mathbf{w}^T$	$\odot$	$\mathbf{u}^T \oplus . \otimes \mathbf{A}$
eWiseMult	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A} \otimes \mathbf{B}$
	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$\mathbf{u} \otimes \mathbf{v}$
eWiseAdd	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A} \oplus \mathbf{B}$
	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$\mathbf{u} \oplus \mathbf{v}$
extract	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A}(\boldsymbol{i}, \boldsymbol{j})$
	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$\mathbf{u}(\boldsymbol{i})$
assign	$\mathbf{C}\langle\mathbf{M}, r\rangle(\boldsymbol{i}, \boldsymbol{j})$	$=$	$\mathbf{C}(\boldsymbol{i}, \boldsymbol{j})$	$\odot$	$\mathbf{A}$
	$\mathbf{w}\langle\mathbf{m}, r\rangle(\boldsymbol{i})$	$=$	$\mathbf{w}(\boldsymbol{i})$	$\odot$	$\mathbf{u}$
reduce (row)	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$[\oplus_j \mathbf{A}(:, j)]$
reduce (scalar)	s	$=$	s	$\odot$	$[\oplus_{i,j} \mathbf{A}(i, j)]$
	s	$=$	s	$\odot$	$[\oplus_i \mathbf{u}(i)]$
apply	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$f_u(\mathbf{A})$
	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$f_u(\mathbf{u})$
apply(indexop)	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s)$
	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), s)$
select	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A}\langle f_i(\mathbf{A}, \mathbf{ind}(\mathbf{A}), s)\rangle$
	$\mathbf{w}\langle\mathbf{m}, r\rangle$	$=$	$\mathbf{w}$	$\odot$	$\mathbf{u}\langle f_i(\mathbf{u}, \mathbf{ind}(\mathbf{u}), s)\rangle$
transpose	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A}^T$
kronecker	$\mathbf{C}\langle\mathbf{M}, r\rangle$	$=$	$\mathbf{C}$	$\odot$	$\mathbf{A} \circledast \mathbf{B}$

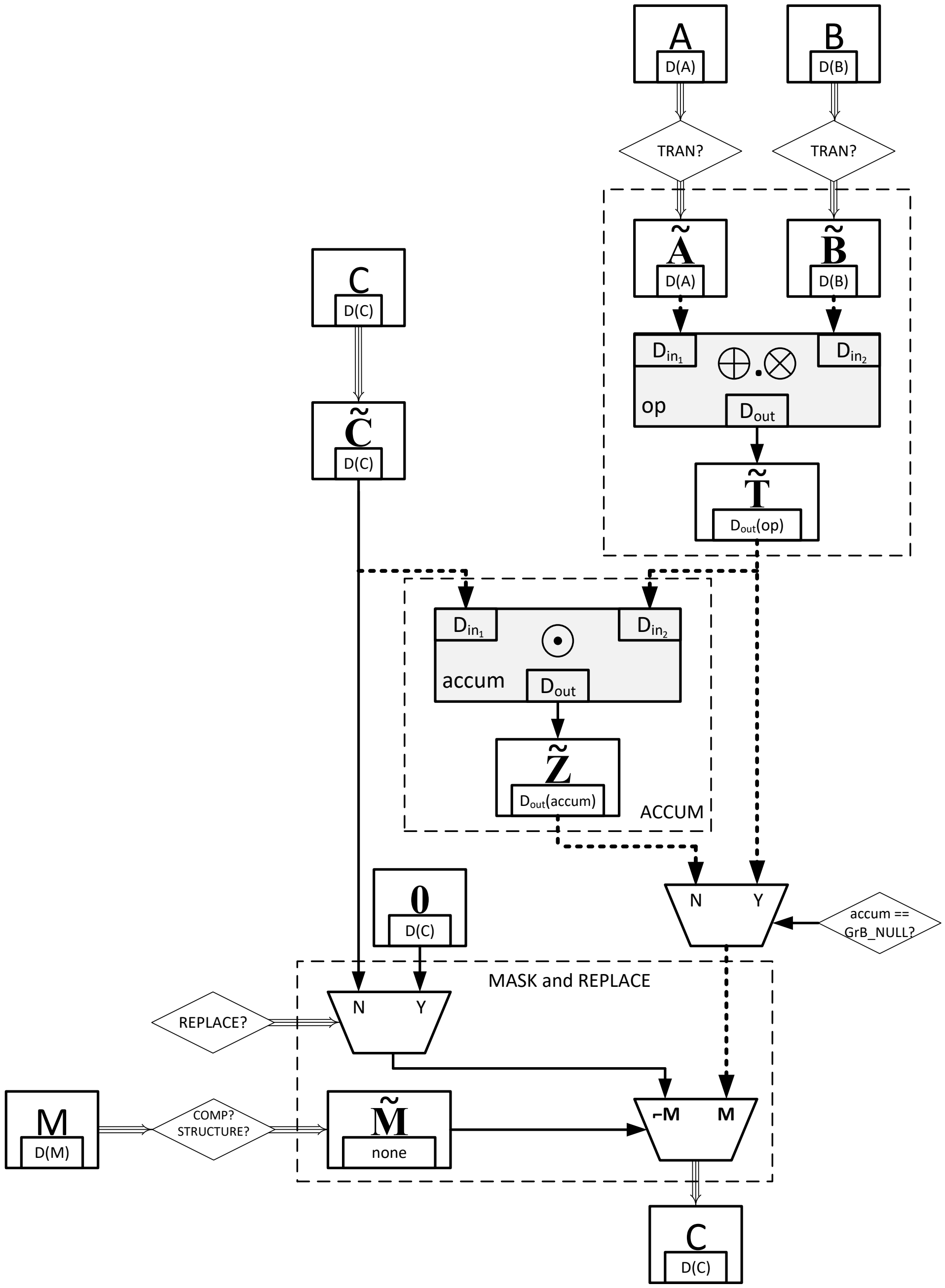

Figure 4.1: Flowchart for the GraphBLAS operations. Although shown specifically for the mxm operation, many elements are common to all operations: such as the "ACCUM" and "MASK and REPLACE" blocks. The triple arrows (⇛) denote where "as if copy" takes place (including both collections and descriptor settings). The bold, dotted arrows indicate where casting may occur between different domains.

argument to a GraphBLAS operation and the associated descriptor indicates the transpose option, then the operation occurs as if on the transposed matrix. In this case, the relationships between the sizes in each dimension shift in the mathematically expected way.

Masks: Structure-only, Complement, and Replace

When a GraphBLAS operation supports the use of an optional mask, that mask is specified through a GraphBLAS vector (for one-dimensional masks) or a GraphBLAS matrix (for two-dimensional masks). When a mask is used and the `GrB_STRUCTURE` descriptor value is not set, it is applied to the result from the operation wherever the stored values in the mask evaluate to true. If the `GrB_STRUCTURE` descriptor is set, the mask is applied to the result from the operation wherever the mask as a stored value (regardless of that value). Wherever the mask is applied, the result from the operation is either assigned to the provided output matrix/vector or, if a binary accumulation operation is provided, the result is accumulated into the corresponding elements of the provided output matrix/vector.

Given a GraphBLAS vector $\mathbf{v} = \langle D, N, \{(i, v_i)\}\rangle$, a one-dimensional mask is derived for use in the operation as follows:

$$\mathbf{m} = \begin{cases} \langle N, \{\mathbf{ind}(\mathbf{v})\}\rangle, & \text{if } \texttt{GrB\_STRUCTURE} \text{ is specified,} \\ \langle N, \{i : \texttt{(bool)}v_i = \mathsf{true}\}\rangle, & \text{otherwise} \end{cases}$$

where $\texttt{(bool)}v_i$ denotes casting the value v_i to a Boolean value (true or false). Likewise, given a GraphBLAS matrix $\mathbf{A} = \langle D, M, N, \{(i, j, A_{ij})\}\rangle$, a two-dimensional mask is derived for use in the operation as follows:

$$\mathbf{M} = \begin{cases} \langle M, N, \{\mathbf{ind}(\mathbf{A})\}\rangle, & \text{if } \texttt{GrB\_STRUCTURE} \text{ is specified,} \\ \langle M, N, \{(i, j) : \texttt{(bool)}A_{ij} = \mathsf{true}\}\rangle, & \text{otherwise} \end{cases}$$

where $\texttt{(bool)}A_{ij}$ denotes casting the value A_{ij} to a Boolean value. (true or false)

In both the one- and two-dimensional cases, the mask may also have a subsequent complement operation applied (*Section* 3.5.4) as specified in the descriptor, before a final mask is generated for use in the operation.

When the descriptor of an operation with a mask has specified that the GrB_REPLACE value is to be applied to the output (GrB_OUTP), then anywhere the mask is not true, the corresponding location in the output is cleared.

Invalid and uninitialized objects

Upon entering a GraphBLAS operation, the first step is a check that all objects are valid and initialized. (Optional parameters can be set to GrB_NULL, which always counts as a valid object.) An invalid object is one that could not be computed due to a previous execution error. An unitialized object is one that has not yet been created by a corresponding new or dup method. Appropriate error codes are returned if an object is not initialized (GrB_UNINITIALIZED_OBJECT) or invalid (GrB_INVALID_OBJECT).

To support the detection of as many cases of uninitialized objects as possible, it is strongly recommended to initialize all GraphBLAS objects to the predefined value GrB_INVALID_HANDLE at the point of their declaration, as shown in the following examples:

```
GrB_Type        type = GrB_INVALID_HANDLE;
GrB_Semiring    semiring = GrB_INVALID_HANDLE;
GrB_Matrix      matrix = GrB_INVALID_HANDLE;
```

Compliance

We follow a *prescriptive* approach to the definition of the semantics of GraphBLAS operations. That is, for each operation we give a recipe for producing its outcome. Any implementation that produces the same outcome, and follows the GraphBLAS execution model (Section 2.5) and error model (Section 2.6) is a conforming implementation.

4.3.1 mxm: Matrix-matrix multiply

Multiplies a matrix with another matrix on a semiring. The result is a matrix.

C Syntax

```
GrB_Info GrB_mxm(GrB_Matrix             C,
                 const GrB_Matrix       Mask,
                 const GrB_BinaryOp     accum,
                 const GrB_Semiring     op,
                 const GrB_Matrix       A,
                 const GrB_Matrix       B,
                 const GrB_Descriptor   desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the matrix product. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring used in the matrix-matrix multiply.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.

B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
B	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_DIMENSION_MISMATCH Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

GrB_mxm computes the matrix product $\mathsf{C} = \mathsf{A} \oplus . \otimes \mathsf{B}$ or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot (\mathsf{A} \oplus . \otimes \mathsf{B})$ (where matrices A and B can be optionally transposed). Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to four argument matrices are used in the GrB_mxm operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$
4. $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$

The argument matrices, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the semiring.
3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the semiring.
4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the semiring.
5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself.

If any compatibility rule above is violated, execution of GrB_mxm ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
      $\{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? $\mathsf{A}^T : \mathsf{A}$.
4. Matrix $\widetilde{\mathbf{B}} \leftarrow$ desc[GrB_INP1].GrB_TRAN ? $\mathsf{B}^T : \mathsf{B}$.

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.
4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{B}})$.
5. $\mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathbf{nrows}(\widetilde{\mathbf{B}})$.

If any compatibility rule above is violated, execution of GrB_mxm ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the matrix multiplication and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the product of matrices $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i,j,T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(:,j)) \neq \emptyset\}\rangle$ is created. The value of each of its elements is computed by

$$T_{ij} = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{B}}(:,j))} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{B}}(k,j)),$$

where $\oplus$ and $\otimes$ are the additive and multiplicative operators of semiring op, respectively.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

  $$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i,j,Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  $$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

  $$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

  $$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.2 vxm: Vector-matrix multiply

Multiplies a (row) vector with a matrix on an semiring. The result is a vector.

C Syntax

```
GrB_Info GrB_vxm(GrB_Vector             w,
                 const GrB_Vector       mask,
                 const GrB_BinaryOp     accum,
                 const GrB_Semiring     op,
                 const GrB_Vector       u,
                 const GrB_Matrix       A,
                 const GrB_Descriptor   desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the vector-matrix product. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) Semiring used in the vector-matrix multiply.

u (IN) The GraphBLAS vector holding the values for the left-hand vector in the multiplication.

A (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
A	GrB_INP1	GrB_TRAN	Use transpose of A for the operation.

Return Values

- **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
- **GrB_PANIC** Unknown internal error.
- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- **GrB_OUT_OF_MEMORY** Not enough memory available for the operation.
- **GrB_UNINITIALIZED_OBJECT** One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).
- **GrB_DIMENSION_MISMATCH** Mask, vector, and/or matrix dimensions are incompatible.
- **GrB_DOMAIN_MISMATCH** The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

GrB_vxm computes the vector-matrix product $\mathsf{w}^T = \mathsf{u}^T \oplus . \otimes \mathsf{A}$, or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{w}^T = \mathsf{w}^T \odot \left(\mathsf{u}^T \oplus . \otimes \mathsf{A}\right)$ (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

- **Setup** The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.
- **Compute** The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors or matrices are used in the GrB_vxm operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$
4. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$

The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the semiring.
3. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the semiring.
4. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the semiring.
5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_vxm ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\, i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.
3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

4. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP1].GrB_TRAN ? A^T : A.

The internal matrices and masks are checked for shape compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$.
2. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.
3. $\mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of GrB_vxm ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the vector-matrix multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the product of vector $\widetilde{\mathbf{u}}^T$ and matrix $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(j, t_j) : \mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{A}}(:, j)) \neq \emptyset\}\rangle$ is created. The value of each of its elements is computed by

$$t_j = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{A}}(:,j))} (\widetilde{\mathbf{u}}(k) \otimes \widetilde{\mathbf{A}}(k, j)),$$

where $\oplus$ and $\otimes$ are the additive and multiplicative operators of semiring op, respectively.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

  $$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

  $$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

  $$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  $$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.3 mxv: Matrix-vector multiply

Multiplies a matrix by a vector on a semiring. The result is a vector.

C Syntax

```
GrB_Info GrB_mxv(GrB_Vector             w,
                 const GrB_Vector       mask,
                 const GrB_BinaryOp     accum,
                 const GrB_Semiring     op,
                 const GrB_Matrix       A,
                 const GrB_Vector       u,
                 const GrB_Descriptor   desc);
```

Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the matrix-vector product. On output, this vector holds the results of the operation.

- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

- **accum** (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
- **op** (IN) Semiring used in the vector-matrix multiply.
- **A** (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.
- **u** (IN) The GraphBLAS vector holding the values for the right-hand vector in the multiplication.
- **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

- **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
- **GrB_PANIC** Unknown internal error.
- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- **GrB_OUT_OF_MEMORY** Not enough memory available for the operation.
- **GrB_UNINITIALIZED_OBJECT** One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for matrix or vector parameters).

GrB_DIMENSION_MISMATCH Mask, vector, and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors/matrices are incompatible with the corresponding domains of the semiring or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

GrB_mxv computes the matrix-vector product $\mathsf{w} = \mathsf{A} \oplus . \otimes \mathsf{u}$, or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{w} = \mathsf{w} \odot (\mathsf{A} \oplus . \otimes \mathsf{u})$ (where matrix A can be optionally transposed). Logically, this operation occurs in three steps:

Setup The internal vectors, matrices and mask used in the computation are formed and their domains/dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors or matrices are used in the GrB_mxv operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$
4. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument matrices, vectors, the semiring, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the semiring.
3. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the semiring.
4. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the semiring.
5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the semiring must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_mxv ends and the domain mismatch error listed above is returned.

From the argument vectors and matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\}\rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If $\mathsf{desc[GrB\_MASK].GrB\_STRUCTURE}$ is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.
   (c) If $\mathsf{desc[GrB\_MASK].GrB\_COMP}$ is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc[GrB\_INP0].GrB\_TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$.
4. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

The internal matrices and masks are checked for shape compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$.
2. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.
3. $\mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of GrB_mxv ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the matrix-vector multiplication and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the product of matrix $\widetilde{\mathbf{A}}$ and vector $\widetilde{\mathbf{u}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(i, t_i) : \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{u}}) \neq \emptyset\}\rangle$ is created. The value of each of its elements is computed by

$$t_i = \bigoplus_{k \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:)) \cap \mathbf{ind}(\widetilde{\mathbf{u}})} (\widetilde{\mathbf{A}}(i,k) \otimes \widetilde{\mathbf{u}}(k)),$$

where $\oplus$ and $\otimes$ are the additive and multiplicative operators of semiring op, respectively.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.4 eWiseMult: Element-wise multiplication

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

4.3.4.1 eWiseMult: Vector variant

Perform element-wise (general) multiplication on the intersection of elements of two vectors, producing a third vector as result.

C Syntax

```
GrB_Info GrB_eWiseMult(GrB_Vector             w,
                       const GrB_Vector       mask,
                       const GrB_BinaryOp     accum,
                       const GrB_Semiring     op,
                       const GrB_Vector       u,
                       const GrB_Vector       v,
                       const GrB_Descriptor   desc);

GrB_Info GrB_eWiseMult(GrB_Vector             w,
                       const GrB_Vector       mask,
                       const GrB_BinaryOp     accum,
                       const GrB_Monoid       op,
                       const GrB_Vector       u,
                       const GrB_Vector       v,
                       const GrB_Descriptor   desc);

GrB_Info GrB_eWiseMult(GrB_Vector             w,
                       const GrB_Vector       mask,
                       const GrB_BinaryOp     accum,
                       const GrB_BinaryOp     op,
                       const GrB_Vector       u,
                       const GrB_Vector       v,
                       const GrB_Descriptor   desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigodot(\mathsf{op}) \rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigodot(\mathsf{op}) \rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigotimes(\mathsf{op}) \rangle$; the additive monoid is ignored.

u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.

v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_DIMENSION_MISMATCH Mask or vector dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_eWiseMult computes the element-wise "product" of two GraphBLAS vectors: $\mathsf{w} = \mathsf{u} \otimes \mathsf{v}$, or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{w} = \mathsf{w} \odot (\mathsf{u} \otimes \mathsf{v})$. Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors are used in the GrB_eWiseMult operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$
4. $\mathsf{v} = \langle \mathbf{D}(\mathsf{v}), \mathbf{size}(\mathsf{v}), \mathbf{L}(\mathsf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
3. $\mathbf{D}(\mathsf{v})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
4. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall\ i : 0 \le i < \mathbf{size}(\mathsf{w})\}\rangle$.
   (b) If mask $\neq$ GrB_NULL,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.
3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.
4. Vector $\widetilde{\mathbf{v}} \leftarrow \mathsf{v}$.

The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}}) = \mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{size}(\widetilde{\mathbf{v}})$.

If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the element-wise "product" and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the element-wise "product" of $\widetilde{\mathbf{u}}$ and vector $\widetilde{\mathbf{v}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}) \neq \emptyset\}\rangle$ is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \otimes \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.4.2 eWiseMult: Matrix variant

Perform element-wise (general) multiplication on the intersection of elements of two matrices, producing a third matrix as result.

C Syntax

```
GrB_Info GrB_eWiseMult(GrB_Matrix            C,
                       const GrB_Matrix       Mask,
                       const GrB_BinaryOp     accum,
                       const GrB_Semiring     op,
                       const GrB_Matrix       A,
                       const GrB_Matrix       B,
                       const GrB_Descriptor   desc);

GrB_Info GrB_eWiseMult(GrB_Matrix            C,
                       const GrB_Matrix       Mask,
                       const GrB_BinaryOp     accum,
                       const GrB_Monoid       op,
                       const GrB_Matrix       A,
                       const GrB_Matrix       B,
                       const GrB_Descriptor   desc);

GrB_Info GrB_eWiseMult(GrB_Matrix            C,
                       const GrB_Matrix       Mask,
                       const GrB_BinaryOp     accum,
                       const GrB_BinaryOp     op,
                       const GrB_Matrix       A,
                       const GrB_Matrix       B,
                       const GrB_Descriptor   desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigodot(\mathsf{op})\rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigodot(\mathsf{op})\rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigotimes(\mathsf{op})\rangle$; the additive monoid is ignored.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the operation.

B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
B	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_DIMENSION_MISMATCH Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_eWiseMult computes the element-wise "product" of two GraphBLAS matrices: $\mathsf{C} = \mathsf{A} \otimes \mathsf{B}$, or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot (\mathsf{A} \otimes \mathsf{B})$. Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to four argument matrices are used in the GrB_eWiseMult operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$
4. $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$

The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
5. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \le i < \mathbf{nrows}(\mathsf{C}), 0 \le j < \mathbf{ncols}(\mathsf{C})\} \rangle$.
   (b) If $\mathsf{Mask} \ne \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
      $\{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A.
4. Matrix $\widetilde{\mathbf{B}} \leftarrow$ desc[GrB_INP1].GrB_TRAN ? B^T : B.

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}) = \mathbf{nrows}(\widetilde{\mathbf{A}}) = \mathbf{nrows}(\widetilde{\mathbf{C}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}) = \mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathbf{ncols}(\widetilde{\mathbf{C}})$.

If any compatibility rule above is violated, execution of GrB_eWiseMult ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the element-wise "product" and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the element-wise product of $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i,j,T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}}) \ne \emptyset\} \rangle$ is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \otimes \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i, j) \odot \widetilde{\mathbf{T}}(i, j), \text{ if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i, j), \text{ if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i, j), \text{ if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.5 eWiseAdd: Element-wise addition

Note: The difference between eWiseAdd and eWiseMult is not about the element-wise operation but how the index sets are treated. eWiseAdd returns an object whose indices are the "union" of the indices of the inputs whereas eWiseMult returns an object whose indices are the "intersection" of the indices of the inputs. In both cases, the passed semiring, monoid, or operator operates on the set of values from the resulting index set.

4.3.5.1 eWiseAdd: Vector variant

Perform element-wise (general) addition on the elements of two vectors, producing a third vector as result.

C Syntax

```
        GrB_Info GrB_eWiseAdd(GrB_Vector             w,
                              const GrB_Vector       mask,
                              const GrB_BinaryOp     accum,
                              const GrB_Semiring     op,
                              const GrB_Vector       u,
                              const GrB_Vector       v,
                              const GrB_Descriptor   desc);

        GrB_Info GrB_eWiseAdd(GrB_Vector             w,
                              const GrB_Vector       mask,
                              const GrB_BinaryOp     accum,
                              const GrB_Monoid       op,
                              const GrB_Vector       u,
                              const GrB_Vector       v,
                              const GrB_Descriptor   desc);

        GrB_Info GrB_eWiseAdd(GrB_Vector             w,
                              const GrB_Vector       mask,
                              const GrB_BinaryOp     accum,
                              const GrB_BinaryOp     op,
                              const GrB_Vector       u,
                              const GrB_Vector       v,
                              const GrB_Descriptor   desc);
```

Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the element-wise operation. On output, this vector holds the results of the operation.

- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigodot(\mathsf{op}) \rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigodot(\mathsf{op}) \rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigoplus(\mathsf{op}) \rangle$; the multiplicative binary op and additive identity are ignored.

u (IN) The GraphBLAS vector holding the values for the left-hand vector in the operation.

v (IN) The GraphBLAS vector holding the values for the right-hand vector in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_DIMENSION_MISMATCH Mask or vector dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_eWiseAdd computes the element-wise "sum" of two GraphBLAS vectors: $\mathsf{w} = \mathsf{u} \oplus \mathsf{v}$, or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{w} = \mathsf{w} \odot (\mathsf{u} \oplus \mathsf{v})$. Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to four argument vectors are used in the GrB_eWiseAdd operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$
4. $\mathsf{v} = \langle \mathbf{D}(\mathsf{v}), \mathbf{size}(\mathsf{v}), \mathbf{L}(\mathsf{v}) = \{(i, v_i)\} \rangle$

The argument vectors, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
3. $\mathbf{D}(\mathsf{v})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
4. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
5. $\mathbf{D}(\mathsf{u})$ and $\mathbf{D}(\mathsf{v})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
6. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.
4. Vector $\widetilde{\mathbf{v}} \leftarrow \mathsf{v}$.

The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}}) = \mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{size}(\widetilde{\mathbf{v}})$.

If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the element-wise "sum" and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the element-wise "sum" of $\widetilde{\mathbf{u}}$ and vector $\widetilde{\mathbf{v}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector $\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, t_i) : \mathbf{ind}(\widetilde{\mathbf{u}}) \cup \mathbf{ind}(\widetilde{\mathbf{v}}) \neq \emptyset\} \rangle$ is created. The value of each of its elements is computed by:

$$t_i = (\widetilde{\mathbf{u}}(i) \oplus \widetilde{\mathbf{v}}(i)), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}}))$$

$$t_i = \widetilde{\mathbf{u}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{u}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

$$t_i = \widetilde{\mathbf{v}}(i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{v}}) - (\mathbf{ind}(\widetilde{\mathbf{u}}) \cap \mathbf{ind}(\widetilde{\mathbf{v}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.5.2 eWiseAdd: Matrix variant

Perform element-wise (general) addition on the elements of two matrices, producing a third matrix as result.

C Syntax

```
GrB_Info GrB_eWiseAdd(GrB_Matrix             C,
                      const GrB_Matrix       Mask,
                      const GrB_BinaryOp     accum,
                      const GrB_Semiring     op,
                      const GrB_Matrix       A,
                      const GrB_Matrix       B,
                      const GrB_Descriptor   desc);

GrB_Info GrB_eWiseAdd(GrB_Matrix             C,
                      const GrB_Matrix       Mask,
                      const GrB_BinaryOp     accum,
                      const GrB_Monoid       op,
                      const GrB_Matrix       A,
                      const GrB_Matrix       B,
                      const GrB_Descriptor   desc);

GrB_Info GrB_eWiseAdd(GrB_Matrix             C,
                      const GrB_Matrix       Mask,
                      const GrB_BinaryOp     accum,
                      const GrB_BinaryOp     op,
                      const GrB_Matrix       A,
                      const GrB_Matrix       B,
                      const GrB_Descriptor   desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the element-wise operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "sum" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \oplus \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigodot(\mathsf{op})\rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigodot(\mathsf{op})\rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigoplus(\mathsf{op})\rangle$; the multiplicative binary op and additive identity are ignored.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the operation.

B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the operation.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
B	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_DIMENSION_MISMATCH Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_eWiseAdd computes the element-wise "sum" of two GraphBLAS matrices: $\mathsf{C} = \mathsf{A} \oplus \mathsf{B}$, or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot (\mathsf{A} \oplus \mathsf{B})$. Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to four argument matrices are used in the GrB_eWiseAdd operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\}\rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\}\rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\}\rangle$
4. $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\}\rangle$

The argument matrices, the "sum" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$.
3. $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$.
4. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
5. $\mathbf{D}(\mathsf{A})$ and $\mathbf{D}(\mathsf{B})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$.
6. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of op must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
      $\{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? $\mathsf{A}^T : \mathsf{A}$.
4. Matrix $\widetilde{\mathbf{B}} \leftarrow$ desc[GrB_INP1].GrB_TRAN ? $\mathsf{B}^T : \mathsf{B}$.

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}}) = \mathbf{nrows}(\widetilde{\mathbf{A}}) = \mathbf{nrows}(\widetilde{\mathbf{C}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}}) = \mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathbf{ncols}(\widetilde{\mathbf{C}})$.

If any compatibility rule above is violated, execution of GrB_eWiseAdd ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the element-wise "sum" and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the element-wise sum of $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \{(i,j,T_{ij}) : \mathbf{ind}(\widetilde{\mathbf{A}}) \cup \mathbf{ind}(\widetilde{\mathbf{B}}) \neq \emptyset\}\rangle$ is created. The value of each of its elements is computed by

$$T_{ij} = (\widetilde{\mathbf{A}}(i,j) \oplus \widetilde{\mathbf{B}}(i,j)), \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})$$

$$T_{ij} = \widetilde{\mathbf{A}}(i,j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{A}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

$$T_{ij} = \widetilde{\mathbf{B}}(i.j), \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{B}}) - (\mathbf{ind}(\widetilde{\mathbf{A}}) \cap \mathbf{ind}(\widetilde{\mathbf{B}})))$$

where the difference operator in the previous expressions refers to set difference.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

  $$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i,j,Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  $$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

  $$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

  $$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.6 extract: Selecting sub-graphs

Extract a subset of a matrix or vector.

4.3.6.1 extract: Standard vector variant

Extract a sub-vector from a larger vector as specified by a set of indices. The result is a vector whose size is equal to the number of indices.

C Syntax

```
GrB_Info GrB_extract(GrB_Vector              w,
                     const GrB_Vector        mask,
                     const GrB_BinaryOp      accum,
                     const GrB_Vector        u,
                     const GrB_Index        *indices,
                     GrB_Index               nindices,
                     const GrB_Descriptor    desc);
```

Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.

- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- u (IN) The GraphBLAS vector from which the subset is extracted.

- indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations of elements from u that are extracted. If all elements of u are to be extracted in order from 0 to nindices -1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.

- nindices (IN) The number of values in indices array. Must be equal to **size**(w).

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_INDEX_OUT_OF_BOUNDS A value in indices is greater than or equal to **size**(u). In non-blocking mode, this error can be deferred.

GrB_DIMENSION_MISMATCH mask and w dimensions are incompatible, or nindices $\neq$ **size**(w).

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_NULL_POINTER Argument row_indices is a NULL pointer.

Description

This variant of GrB_extract computes the result of extracting a subset of locations from a GraphBLAS vector in a specific order: w = u(indices); or, if an optional binary accumulation operator

($\odot$) is provided, $\mathsf{w} = \mathsf{w} \odot \mathsf{u}(\mathsf{indices})$. More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \qquad\qquad \mathsf{u}(\mathsf{indices}[i]),\ \forall\ i:\ 0 \leq i < \mathsf{nindices}, \quad \text{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{u}(\mathsf{indices}[i]),\ \forall\ i:\ 0 \leq i < \mathsf{nindices} \end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB_extract operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}(\mathsf{u})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_extract ends and the domain mismatch error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.

(b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,

i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,

ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.

(c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.

3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

4. The internal index array, $\widetilde{\boldsymbol{I}}$, is computed from argument indices as follows:

   (a) If $\mathsf{indices} = \mathsf{GrB\_ALL}$, then $\widetilde{\boldsymbol{I}}[i] = i, \ \forall\ i : 0 \leq i < \mathsf{nindices}$.

   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{indices}[i], \ \forall\ i : 0 \leq i < \mathsf{nindices}$.

The internal vectors and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$

2. $\mathsf{nindices} = \mathbf{size}(\widetilde{\mathbf{w}})$.

If any compatibility rule above is violated, execution of GrB_extract ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the extraction from $\widetilde{\mathbf{u}}$ in their destination locations relative to $\widetilde{\mathbf{w}}$.

- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, \widetilde{\mathbf{u}}(\widetilde{\boldsymbol{I}}[i]))\ \forall\ i, 0 \leq i < \mathsf{nindices} : \widetilde{\boldsymbol{I}}[i] \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle.$$

At this point, if any value in $\widetilde{\boldsymbol{I}}$ is not in the valid range of indices for vector $\widetilde{\mathbf{u}}$, the execution of GrB_extract ends and the index-out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.

- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.6.2 extract: Standard matrix variant

Extract a sub-matrix from a larger matrix as specified by a set of row indices and a set of column indices. The result is a matrix whose size is equal to size of the sets of indices.

C Syntax

```
GrB_Info GrB_extract(GrB_Matrix            C,
                     const GrB_Matrix      Mask,
                     const GrB_BinaryOp    accum,
                     const GrB_Matrix      A,
                     const GrB_Index      *row_indices,
                     GrB_Index             nrows,
                     const GrB_Index      *col_indices,
                     GrB_Index             ncols,
                     const GrB_Descriptor  desc);
```

Parameters

- **C** (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the extract operation. On output, the matrix holds the results of the operation.

- **Mask** (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

- **accum** (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- **A** (IN) The GraphBLAS matrix from which the subset is extracted.

- **row_indices** (IN) Pointer to the ordered set (array) of indices corresponding to the rows of A from which elements are extracted. If elements in all rows of A are to be extracted in order, GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.

- **nrows** (IN) The number of values in the row_indices array. Must be equal to **nrows**(C).

- **col_indices** (IN) Pointer to the ordered set (array) of indices corresponding to the columns of A from which elements are extracted. If elements in all columns of A are to be extracted in order, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.

- **ncols** (IN) The number of values in the col_indices array. Must be equal to **ncols**(C).

- **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for the operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).
GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to **nrows**(A), or a value in col_indices is greater than or equal to **ncols**(A). In non-blocking mode, this error can be deferred.
GrB_DIMENSION_MISMATCH	Mask and C dimensions are incompatible, nrows $\neq$ **nrows**(C), or ncols $\neq$ **ncols**(C).
GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_NULL_POINTER	Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

Description

This variant of GrB_extract computes the result of extracting a subset of locations from specified rows and columns of a GraphBLAS matrix in a specific order: $\mathsf{C} = \mathsf{A}(\mathsf{row\_indices}, \mathsf{col\_indices})$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot \mathsf{A}(\mathsf{row\_indices}, \mathsf{col\_indices})$. More explicitly (not accounting for an optional transpose of A):

$$\begin{aligned} \mathsf{C}(i,j) &= \phantom{\mathsf{C}(i,j) \odot{}} \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \;\forall\; i,j \;:\; 0 \le i < \mathsf{nrows},\; 0 \le j < \mathsf{ncols}, \text{ or} \\ \mathsf{C}(i,j) &= \mathsf{C}(i,j) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_indices}[j]) \;\forall\; i,j \;:\; 0 \le i < \mathsf{nrows},\; 0 \le j < \mathsf{ncols} \end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_extract operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\}\rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\}\rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\}\rangle$

The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{A})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_extract ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, mask, and index arrays used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i, j), \forall i, j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
      $\{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i, j) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? $\mathsf{A}^T : \mathsf{A}$.

4. The internal row index array, $\widetilde{\boldsymbol{I}}$, is computed from argument row_indices as follows:

   (a) If row_indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \forall i : 0 \leq i <$ nrows.

   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] =$ row_indices$[i], \forall i : 0 \leq i <$ nrows.

5. The internal column index array, $\widetilde{\boldsymbol{J}}$, is computed from argument col_indices as follows:

   (a) If col_indices = GrB_ALL, then $\widetilde{\boldsymbol{J}}[j] = j, \forall j : 0 \leq j <$ ncols.

   (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] =$ col_indices$[j], \forall j : 0 \leq j <$ ncols.

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) =$ nrows.
4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) =$ ncols.

If any compatibility rule above is violated, execution of GrB_extract ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the extraction from $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}),\\ \{(i, j, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j])) \ \forall \ (i, j), \ 0 \leq i < \mathsf{nrows}, \ 0 \leq j < \mathsf{ncols} : (\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j]) \in \mathbf{ind}(\widetilde{\mathbf{A}})\} \rangle.$$

At this point, if any value in the $\widetilde{\boldsymbol{I}}$ array is not in the range $[0, \ \mathbf{nrows}(\widetilde{\mathbf{A}}))$ or any value in the $\widetilde{\boldsymbol{J}}$ array is not in the range $[0, \ \mathbf{ncols}(\widetilde{\mathbf{A}}))$, the execution of GrB_extract ends and the index out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix C is invalid from this point forward in the sequence.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.

- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i,j,Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.6.3 extract: Column (and row) variant

Extract from one column of a matrix into a vector. Note that with the transpose descriptor for the source matrix, elements of an arbitrary row of the matrix can be extracted with this function as well.

C Syntax

```
GrB_Info GrB_extract(GrB_Vector              w,
                     const GrB_Vector        mask,
                     const GrB_BinaryOp      accum,
                     const GrB_Matrix        A,
                     const GrB_Index        *row_indices,
                     GrB_Index               nrows,
                     GrB_Index               col_index,
                     const GrB_Descriptor    desc);
```

Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the extract operation. On output, this vector holds the results of the operation.

- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- A (IN) The GraphBLAS matrix from which the column subset is extracted.

- row_indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations within the specified column of A from which elements are extracted. If elements in all rows of A are to be extracted in order, GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation.

- nrows (IN) The number of indices in the row_indices array. Must be equal to **size**(w).

- col_index (IN) The index of the column of A from which to extract values. It must be in the range [0, **ncols**(A)).

- desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).

GrB_INVALID_INDEX col_index is outside the allowable range (i.e., greater than **ncols**(A)).

GrB_INDEX_OUT_OF_BOUNDS A value in row_indices is greater than or equal to **nrows**(A). In non-blocking mode, this error can be deferred.

GrB_DIMENSION_MISMATCH mask and w dimensions are incompatible, or nrows $\neq$ **size**(w).

GrB_DOMAIN_MISMATCH The domains of the vector or matrix are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_NULL_POINTER Argument row_indices is a NULL pointer.

Description

This variant of GrB_extract computes the result of extracting a subset of locations (in a specific order) from a specified column of a GraphBLAS matrix: $\mathsf{w} = \mathsf{A}(:, \mathsf{col\_index})(\mathsf{row\_indices})$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{w} = \mathsf{w} \odot \mathsf{A}(:, \mathsf{col\_index})(\mathsf{row\_indices})$. More explicitly:

$$\begin{aligned} \mathsf{w}(i) &= \qquad\quad \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \ \forall\ i:\ 0 \leq i < \mathsf{nrows}, \quad \text{or} \\ \mathsf{w}(i) &= \mathsf{w}(i) \odot \mathsf{A}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \ \forall\ i:\ 0 \leq i < \mathsf{nrows} \end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal matrices, vectors, and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors and matrices are used in this GrB_extract operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\}\rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\}\rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\}\rangle$

The argument vectors, matrix and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}(\mathsf{A})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_extract ends and the domain mismatch error listed above is returned.

From the arguments, the internal vector, matrix, mask, and index array used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\}\rangle$.

   (b) If mask $\neq$ GrB_NULL,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.

3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A.

4. The internal row index array, $\widetilde{\boldsymbol{I}}$, is computed from argument row_indices as follows:
   (a) If indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \ \forall\ i : 0 \leq i < \mathsf{nrows}$.
   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{indices}[i], \ \forall\ i : 0 \leq i < \mathsf{nrows}$.

The internal vector, mask, and index array are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$
2. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathsf{nrows}$.

If any compatibility rule above is violated, execution of GrB_extract ends and the dimension mismatch error listed above is returned.

The col_index parameter is checked for a valid value. The following condition must hold:

1. $0 \ \leq\ \mathsf{col\_index} \ <\ \mathbf{ncols}(\mathsf{A})$

If the rule above is violated, execution of GrB_extract ends and the invalid index error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the extract and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the extraction from a column of $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{A}), \mathsf{nrows}, \{(i, \widetilde{\mathbf{A}}(\widetilde{\boldsymbol{I}}[i], \mathsf{col\_index}))\ \forall\ i, 0 \leq i < \mathsf{nrows} : (\widetilde{\boldsymbol{I}}[i], \mathsf{col\_index}) \in \mathbf{ind}(\widetilde{\mathbf{A}})\}\rangle.$$

At this point, if any value in $\widetilde{\boldsymbol{I}}$ is not in the range $[0,\ \mathbf{nrows}(\widetilde{\mathbf{A}}))$, the execution of GrB_extract ends and the index-out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.7 assign: Modifying sub-graphs

Assign the contents of a subset of a matrix or vector.

4.3.7.1 assign: Standard vector variant

Assign values from one GraphBLAS vector to a subset of a vector as specified by a set of indices. The size of the input vector is the same size as the index array provided.

C Syntax

```
GrB_Info GrB_assign(GrB_Vector             w,
                    const GrB_Vector      mask,
                    const GrB_BinaryOp    accum,
                    const GrB_Vector      u,
                    const GrB_Index      *indices,
                    GrB_Index             nindices,
                    const GrB_Descriptor  desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

u (IN) The GraphBLAS vector whose contents are assigned to a subset of w.

indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0 to nindices − 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.

nindices (IN) The number of values in indices array. Must be equal to **size**(u).

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
GrB_INDEX_OUT_OF_BOUNDS	A value in indices is greater than or equal to **size**(w). In non-blocking mode, this can be reported as an execution error.
GrB_DIMENSION_MISMATCH	mask and w dimensions are incompatible, or nindices $\neq$ **size**(u).
GrB_DOMAIN_MISMATCH	The domains of the various vectors are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_NULL_POINTER	Argument indices is a NULL pointer.

Description

This variant of GrB_assign computes the result of assigning elements from a source GraphBLAS vector to a destination GraphBLAS vector in a specific order: w(indices) = u; or, if an optional binary accumulation operator ($\odot$) is provided, w(indices) = w(indices) $\odot$ u. More explicitly:

$$\begin{aligned} \mathsf{w}(\mathsf{indices}[i]) &= \mathsf{u}(i),\ \forall\ i\ :\ 0 \leq i < \mathsf{nindices}, \quad \text{or} \\ \mathsf{w}(\mathsf{indices}[i]) &= \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{u}(i),\ \forall\ i\ :\ 0 \leq i < \mathsf{nindices}. \end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in the GrB_assign operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument vectors and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}(\mathsf{u})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal vectors, mask and index array used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall\, i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.

3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

4. The internal index array, $\widetilde{\boldsymbol{I}}$, is computed from argument indices as follows:

   (a) If indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \ \forall\ i : 0 \leq i < \mathsf{nindices}$.

   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{indices}[i], \ \forall\ i : 0 \leq i < \mathsf{nindices}$.

The internal vector and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$

2. $\mathsf{nindices} = \mathbf{size}(\widetilde{\mathbf{u}})$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the elements from $\widetilde{\mathbf{u}}$ in their destination locations relative to $\widetilde{\mathbf{w}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\mathbf{u}}(i)) \forall i, 0 \leq i < \mathsf{nindices} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle.$$

At this point, if any value of $\widetilde{\boldsymbol{I}}[i]$ is outside the valid range of indices for vector $\widetilde{\mathbf{w}}$, computation ends and the method returns the index-out-of-bounds error listed above. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The above expression defines the structure of vector $\widetilde{\mathbf{z}}$ as follows: We start with the structure of $\widetilde{\mathbf{w}}$ ($\mathbf{ind}(\widetilde{\mathbf{w}})$) and remove from it all the indices of $\widetilde{\mathbf{w}}$ that are in the set of indices being assigned ($\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}})$). Finally, we add the structure of $\widetilde{\mathbf{t}}$ ($\mathbf{ind}(\widetilde{\mathbf{t}})$).

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i), \ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \ \text{if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

  where the difference operator refers to set difference.

- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i),\ \text{if}\ i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i),\ \text{if}\ i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i),\ \text{if}\ i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.2 assign: Standard matrix variant

Assign values from one GraphBLAS matrix to a subset of a matrix as specified by a set of indices. The dimensions of the input matrix are the same size as the row and column index arrays provided.

C Syntax

```
GrB_Info GrB_assign(GrB_Matrix             C,
                    const GrB_Matrix       Mask,
                    const GrB_BinaryOp     accum,
                    const GrB_Matrix       A,
```

```
                                const GrB_Index      *row_indices,
                                GrB_Index             nrows,
                                const GrB_Index      *col_indices,
                                GrB_Index             ncols,
                                const GrB_Descriptor  desc);
```

Parameters

- C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, the matrix holds the results of the operation.

- Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- A (IN) The GraphBLAS matrix whose contents are assigned to a subset of C.

- row_indices (IN) Pointer to the ordered set (array) of indices corresponding to the rows of C that are assigned. If all rows of C are to be assigned in order from 0 to nrows $-$ 1, then GrB_ALL can be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.

- nrows (IN) The number of values in the row_indices array. Must be equal to **nrows**(A) if A is not tranposed, or equal to **ncols**(A) if A is transposed.

- col_indices (IN) Pointer to the ordered set (array) of indices corresponding to the columns of C that are assigned. If all columns of C are to be assigned in order from 0 to ncols $-$ 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies assignment of more than one value to the same location which leads to undefined results.

- ncols (IN) The number of values in col_indices array. Must be equal to **ncols**(A) if A is not tranposed, or equal to **nrows**(A) if A is transposed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_INDEX_OUT_OF_BOUNDS A value in row_indices is greater than or equal to **nrows**(C), or a value in col_indices is greater than or equal to **ncols**(C). In non-blocking mode, this can be reported as an execution error.

GrB_DIMENSION_MISMATCH Mask and C dimensions are incompatible, nrows $\neq$ **nrows**(A), or ncols $\neq$ **ncols**(A).

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_NULL_POINTER Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

Description

This variant of GrB_assign computes the result of assigning the contents of A to a subset of rows and columns in C in a specified order: C(row_indices, col_indices) = A; or, if an optional binary accumulation operator ($\odot$) is provided, C(row_indices, col_indices) = C(row_indices, col_indices) $\odot$ A. More explicitly (not accounting for an optional transpose of A):

$$\begin{aligned}
\mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) &= \mathsf{A}(i,j),\ \forall\ i,j\ :\ 0 \le i < \mathsf{nrows},\ 0 \le j < \mathsf{ncols}, \text{ or} \\
\mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) &= \mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) \odot \mathsf{A}(i,j), \\
&\qquad \forall\ (i,j)\ :\ 0 \le i < \mathsf{nrows},\ 0 \le j < \mathsf{ncols}
\end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_assign operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij})\}\rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i,j,M_{ij})\}\rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i,j,A_{ij})\}\rangle$

The argument matrices and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}$(Mask) must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}$(C) must be compatible with $\mathbf{D}$(A).
3. If accum is not GrB_NULL, then $\mathbf{D}$(C) must be compatible with $\mathbf{D}_{in_1}$(accum) and $\mathbf{D}_{out}$(accum) of the accumulation operator and $\mathbf{D}$(A) must be compatible with $\mathbf{D}_{in_2}$(accum) of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, mask, and index arrays used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

2. Two-dimensional mask $\widetilde{\mathbf{M}}$ is computed from argument Mask as follows:

   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.

   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,

      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,

      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
      $\{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.

   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.

3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A.

4. The internal row index array, $\widetilde{\boldsymbol{I}}$, is computed from argument row_indices as follows:

   (a) If row_indices = GrB_ALL, then $\widetilde{\boldsymbol{I}}[i] = i, \forall i : 0 \leq i <$ nrows.

   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{row\_indices}[i], \forall i : 0 \leq i <$ nrows.

5. The internal column index array, $\widetilde{\boldsymbol{J}}$, is computed from argument col_indices as follows:

   (a) If col_indices = GrB_ALL, then $\widetilde{\boldsymbol{J}}[j] = j, \forall j : 0 \leq j <$ ncols.

   (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j],\ \forall\ j : 0 \leq j <$ ncols.

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.

2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.

3. $\mathbf{nrows}(\widetilde{\mathbf{A}}) = \mathsf{nrows}$.

4. $\mathbf{ncols}(\widetilde{\mathbf{A}}) = \mathsf{ncols}$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{T}}$: The matrix holding the contents from $\widetilde{\mathbf{A}}$ in their destination locations relative to $\widetilde{\mathbf{C}}$.

- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}),$$
$$\{(\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \widetilde{\mathbf{A}}(i,j)) \ \forall \ (i,j),\ 0 \leq i < \mathsf{nrows},\ 0 \leq j < \mathsf{ncols} : (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})\}\rangle.$$

At this point, if any value in the $\widetilde{\boldsymbol{I}}$ array is not in the range $[0,\ \mathbf{nrows}(\widetilde{\mathbf{C}}))$ or any value in the $\widetilde{\boldsymbol{J}}$ array is not in the range $[0,\ \mathbf{ncols}(\widetilde{\mathbf{C}}))$, the execution of GrB_assign ends and the index out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix C is invalid from this point forward in the sequence.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}}$ is defined as

$$\begin{aligned}\widetilde{\mathbf{Z}} \quad = \quad & \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \\ & \{(i,j,Z_{ij}) \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.\end{aligned}$$

  The above expression defines the structure of matrix $\widetilde{\mathbf{Z}}$ as follows: We start with the structure of $\widetilde{\mathbf{C}}$ ($\mathbf{ind}(\widetilde{\mathbf{C}})$) and remove from it all the indices of $\widetilde{\mathbf{C}}$ that are in the set of indices being assigned ($\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$). Finally, we add the structure of $\widetilde{\mathbf{T}}$ ($\mathbf{ind}(\widetilde{\mathbf{T}})$).

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j),\ \text{if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

  where the difference operator refers to set difference.

- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i,j,Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.3 assign: Column variant

Assign the contents a vector to a subset of elements in one column of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a row of a matrix.

C Syntax

```
GrB_Info GrB_assign(GrB_Matrix            C,
                    const GrB_Vector      mask,
                    const GrB_BinaryOp    accum,
                    const GrB_Vector      u,
                    const GrB_Index      *row_indices,
                    GrB_Index             nrows,
                    GrB_Index             col_index,
                    const GrB_Descriptor  desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the specified column of the output matrix C. The mask dimensions must match those of a single column of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined “built-in” types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a column of C), GrB_NULL should be specified.

- **accum** (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
- **u** (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a column of C.
- **row_indices** (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified column of C that are to be assigned. If all elements of the column in C are to be assigned in order from index 0 to nrows $-$ 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.
- **nrows** (IN) The number of values in row_indices array. Must be equal to **size**(u).
- **col_index** (IN) The index of the column in C to assign. Must be in the range $[0, \mathbf{ncols}(\mathsf{C}))$.
- **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output column in C is cleared (all elements removed) before result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

- **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
- **GrB_PANIC** Unknown internal error.

GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).
GrB_INVALID_INDEX	col_index is outside the allowable range (i.e., greater than **ncols**(C)).
GrB_INDEX_OUT_OF_BOUNDS	A value in row_indices is greater than or equal to **nrows**(C). In non-blocking mode, this can be reported as an execution error.
GrB_DIMENSION_MISMATCH	mask size and number of rows in C are not the same, or nrows $\neq$ **size**(u).
GrB_DOMAIN_MISMATCH	The domains of the matrix and vector are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_NULL_POINTER	Argument row_indices is a NULL pointer.

Description

This variant of GrB_assign computes the result of assigning a subset of locations in a column of a GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector: $\mathsf{C}(:, \mathsf{col\_index}) = \mathsf{u}$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C}(:, \mathsf{col\_index}) = \mathsf{C}(:, \mathsf{col\_index}) \odot \mathsf{u}$. Taking order of row_indices into account, it is more explicitly written as:

$$\begin{aligned} &\mathsf{C}(\mathsf{row\_indices}[i], \mathsf{col\_index}) = \mathsf{u}(i),\ \forall\ i\ :\ 0 \leq i < \mathsf{nrows},\ \text{or} \\ &\mathsf{C}(\mathsf{row\_indices}[i], \mathsf{col\_index}) = \mathsf{C}(\mathsf{row\_indices}[i], \mathsf{col\_index}) \odot \mathsf{u}(i),\ \forall\ i\ :\ 0 \leq i < \mathsf{nrows}. \end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument vectors and matrices are used in this GrB_assign operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)

3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{u})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

The col_index parameter is checked for a valid value. The following condition must hold:

1. $0 \ \le \ \mathsf{col\_index} \ < \ \mathbf{ncols}(\mathsf{C})$

If the rule above is violated, execution of GrB_assign ends and the invalid index error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ($\leftarrow$ denotes copy):

1. The vector, $\widetilde{\mathbf{c}}$, is extracted from a column of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \{(i, C_{ij}) \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C}), j = \mathsf{col\_index}, (i,j) \in \mathbf{ind}(\mathsf{C})\} \rangle$$

2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{nrows}(\mathsf{C}), \{i, \ \forall \ i : 0 \le i < \mathbf{nrows}(\mathsf{C})\} \rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.
4. The internal row index array, $\widetilde{\boldsymbol{I}}$, is computed from argument row_indices as follows:
   (a) If $\mathsf{row\_indices} = \mathsf{GrB\_ALL}$, then $\widetilde{\boldsymbol{I}}[i] = i, \ \forall \ i : 0 \le i < \mathsf{nrows}$.

(b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{row\_indices}[i],\ \forall\ i : 0 \leq i < \mathsf{nrows}$.

The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{c}}) = \mathbf{size}(\widetilde{\mathbf{m}})$
2. $\mathsf{nrows} = \mathbf{size}(\widetilde{\mathbf{u}})$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the elements from $\widetilde{\mathbf{u}}$ in their destination locations relative to $\widetilde{\mathbf{c}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\mathbf{u}}(i))\ \forall\ i,\ 0 \leq i < \mathsf{nrows} : i \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle.$$

At this point, if any value of $\widetilde{\boldsymbol{I}}[i]$ is outside the valid range of indices for vector $\widetilde{\mathbf{c}}$, computation ends and the method returns the index out-of-bounds error listed above. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The above expression defines the structure of vector $\widetilde{\mathbf{z}}$ as follows: We start with the structure of $\widetilde{\mathbf{c}}$ ($\mathbf{ind}(\widetilde{\mathbf{c}})$) and remove from it all the indices of $\widetilde{\mathbf{c}}$ that are in the set of indices being assigned ($\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}})$). Finally, we add the structure of $\widetilde{\mathbf{t}}$ ($\mathbf{ind}(\widetilde{\mathbf{t}})$).

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{c}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{c}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

  where the difference operator refers to set difference.

- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\} \rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{c}}(i) \odot \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$

$$z_i = \widetilde{\mathbf{c}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up the $\widetilde{\mathbf{z}}$ vector are written into the column of the final result matrix, $\mathsf{C}(:, \mathsf{col\_index})$. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in $\mathsf{C}(:, \mathsf{col\_index})$ on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : j \neq \mathsf{col\_index}\} \cup \{(i, \mathsf{col\_index}, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the column of the final result matrix, $\mathsf{C}(:, \mathsf{col\_index})$, and elements of this column that fall outside the set indicated by the mask are unchanged:

$$\begin{aligned} \mathbf{L}(\mathsf{C}) \quad = \quad & \{(i, j, C_{ij}) : j \neq \mathsf{col\_index}\} \cup \\ & \{(i, \mathsf{col\_index}, \widetilde{\mathbf{c}}(i)) : i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \\ & \{(i, \mathsf{col\_index}, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}. \end{aligned}$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.4 assign: Row variant

Assign the contents a vector to a subset of elements in one row of a matrix. Note that since the output cannot be transposed, a different variant of assign is provided to assign to a column of a matrix.

C Syntax

```
GrB_Info GrB_assign(GrB_Matrix             C,
                    const GrB_Vector       mask,
                    const GrB_BinaryOp     accum,
                    const GrB_Vector       u,
                    GrB_Index              row_index,
                    const GrB_Index       *col_indices,
                    GrB_Index              ncols,
                    const GrB_Descriptor   desc);
```

Parameters

- **C** (INOUT) An existing GraphBLAS Matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, this matrix holds the results of the operation.

- **mask** (IN) An optional "write" mask that controls which results from this operation are stored into the specified row of the output matrix C. The mask dimensions must match those of a single row of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of a row of C), GrB_NULL should be specified.

- **accum** (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- **u** (IN) The GraphBLAS vector whose contents are assigned to (a subset of) a row of C.

- **row_index** (IN) The index of the row in C to assign. Must be in the range $[0, \mathbf{nrows}(\mathsf{C}))$.

- **col_indices** (IN) Pointer to the ordered set (array) of indices corresponding to the locations in the specified row of C that are to be assigned. If all elements of the row in C are to be assigned in order from index 0 to $\mathsf{ncols} - 1$, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. If this array contains duplicate values, it implies in assignment of more than one value to the same location which leads to undefined results.

- **ncols** (IN) The number of values in col_indices array. Must be equal to $\mathbf{size}(\mathsf{u})$.

- **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output row in C is cleared (all elements removed) before result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector or matrix parameters).
GrB_INVALID_INDEX	row_index is outside the allowable range (i.e., greater than **nrows**(C)).
GrB_INDEX_OUT_OF_BOUNDS	A value in col_indices is greater than or equal to **ncols**(C). In non-blocking mode, this can be reported as an execution error.
GrB_DIMENSION_MISMATCH	mask size and number of columns in C are not the same, or ncols $\neq$ **size**(u).
GrB_DOMAIN_MISMATCH	The domains of the matrix and vector are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_NULL_POINTER	Argument col_indices is a NULL pointer.

Description

This variant of GrB_assign computes the result of assigning a subset of locations in a row of a GraphBLAS matrix (in a specific order) from the contents of a GraphBLAS vector:

$\mathsf{C}(\mathsf{row\_index}, :) = \mathsf{u}$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C}(\mathsf{row\_index}, :) = \mathsf{C}(\mathsf{row\_index}, :) \odot \mathsf{u}$. Taking order of col_indices into account it is more explicitly written as:

$$\mathsf{C}(\mathsf{row\_index}, \mathsf{col\_indices}[j]) = \mathsf{u}(j),\ \forall\ j\ :\ 0 \leq j < \mathsf{ncols},\ \text{or}$$
$$\mathsf{C}(\mathsf{row\_index}, \mathsf{col\_indices}[j]) = \mathsf{C}(\mathsf{row\_index}, \mathsf{col\_indices}[j]) \odot \mathsf{u}(j),\ \forall\ j\ :\ 0 \leq j < \mathsf{ncols}$$

Logically, this operation occurs in three steps:

Setup The internal matrices, vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument vectors and matrices are used in this GrB_assign operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument vectors, matrix, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{u})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

The row_index parameter is checked for a valid value. The following condition must hold:

1. $0\ \leq\ \mathsf{row\_index}\ <\ \mathbf{nrows}(\mathsf{C})$

If the rule above is violated, execution of GrB_assign ends and the invalid index error listed above is returned.

From the arguments, the internal vectors, mask, and index array used in the computation are formed ($\leftarrow$ denotes copy):

1. The vector, $\widetilde{\mathbf{c}}$, is extracted from a row of C as follows:

$$\widetilde{\mathbf{c}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(j, C_{ij})\ \forall\ j : 0 \leq j < \mathbf{ncols}(\mathsf{C}), i = \mathsf{row\_index}, (i, j) \in \mathbf{ind}(\mathsf{C})\}\rangle$$

2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{ncols}(\mathsf{C}), \{i,\ \forall\ i : 0 \leq i < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.
3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.
4. The internal column index array, $\widetilde{\boldsymbol{J}}$, is computed from argument col_indices as follows:
   (a) If $\mathsf{col\_indices} = \mathsf{GrB\_ALL}$, then $\widetilde{\boldsymbol{J}}[j] = j,\ \forall\ j : 0 \leq j < \mathsf{ncols}$.
   (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j],\ \forall\ j : 0 \leq j < \mathsf{ncols}$.

The internal vectors, matrices, and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{c}}) = \mathbf{size}(\widetilde{\mathbf{m}})$
2. $\mathsf{ncols} = \mathbf{size}(\widetilde{\mathbf{u}})$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the elements from $\widetilde{\mathbf{u}}$ in their destination locations relative to $\widetilde{\mathbf{c}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(\widetilde{\boldsymbol{J}}[j], \widetilde{\mathbf{u}}(j))\ \forall\ j,\ 0 \leq j < \mathsf{ncols} : j \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle.$$

At this point, if any value of $\widetilde{\boldsymbol{J}}[j]$ is outside the valid range of indices for vector $\widetilde{\mathbf{c}}$, computation ends and the method returns the index out-of-bounds error listed above. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix, C, is invalid from this point forward in the sequence.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{C}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The above expression defines the structure of vector $\widetilde{\mathbf{z}}$ as follows: We start with the structure of $\widetilde{\mathbf{c}}$ ($\mathbf{ind}(\widetilde{\mathbf{c}})$) and remove from it all the indices of $\widetilde{\mathbf{c}}$ that are in the set of indices being assigned ($\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}})$). Finally, we add the structure of $\widetilde{\mathbf{t}}$ ($\mathbf{ind}(\widetilde{\mathbf{t}})$).

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{c}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{c}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

  where the difference operator refers to set difference.

- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{c}}), \{(j, z_j)\ \forall\ j \in \mathbf{ind}(\widetilde{\mathbf{c}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_j = \widetilde{\mathbf{c}}(j) \odot \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}})),$$

$$z_j = \widetilde{\mathbf{c}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

$$z_j = \widetilde{\mathbf{t}}(j), \text{ if } j \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{c}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up the $\widetilde{\mathbf{z}}$ vector are written into the column of the final result matrix, $\mathsf{C}(\mathsf{row\_index}, :)$. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in $\mathsf{C}(\mathsf{row\_index}, :)$ on input to this operation are deleted and the new contents of the column is given by:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : i \neq \mathsf{row\_index}\} \cup \{(\mathsf{row\_index}, j, z_j) : j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the column of the final result matrix, $\mathsf{C}(\mathsf{row\_index}, :)$, and elements of this column that fall outside the set indicated by the mask are unchanged:

$$\begin{aligned} \mathbf{L}(\mathsf{C}) \quad = \quad & \{(i, j, C_{ij}) : i \neq \mathsf{row\_index}\} \cup \\ & \{(\mathsf{row\_index}, j, \widetilde{\mathbf{c}}(j)) : j \in (\mathbf{ind}(\widetilde{\mathbf{c}}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \\ & \{(\mathsf{row\_index}, j, z_j) : j \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}. \end{aligned}$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed; however, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.5 assign: Constant vector variant

Assign the same value to a specified subset of vector elements. With the use of GrB_ALL, the entire destination vector can be filled with the constant.

C Syntax

```
GrB_Info GrB_assign(GrB_Vector            w,
                    const GrB_Vector      mask,
                    const GrB_BinaryOp    accum,
                    <type>                val,
                    const GrB_Index      *indices,
                    GrB_Index             nindices,
                    const GrB_Descriptor  desc);

GrB_Info GrB_assign(GrB_Vector            w,
                    const GrB_Vector      mask,
                    const GrB_BinaryOp    accum,
                    const GrB_Scalar      s,
                    const GrB_Index      *indices,
                    GrB_Index             nindices,
                    const GrB_Descriptor  desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the assign operation. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

val (IN) Scalar value to assign to (a subset of) w.

s (IN) Scalar value to assign to (a subset of) w.

indices (IN) Pointer to the ordered set (array) of indices corresponding to the locations in w that are to be assigned. If all elements of w are to be assigned in order from 0 to nindices $-$ 1, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. In this variant, the specific order of the values in the array has no effect on the result. Unlike other variants, if there are duplicated values in this array the result is still defined.

nindices (IN) The number of values in indices array. Must be in the range: $[0, \mathbf{size}(\mathsf{w})]$. If nindices is zero, the operation becomes a NO-OP.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_INDEX_OUT_OF_BOUNDS A value in indices is greater than or equal to $\mathbf{size}(\mathsf{w})$. In non-blocking mode, this can be reported as an execution error.

GrB_DIMENSION_MISMATCH mask and w dimensions are incompatible, or nindices is not less than $\mathbf{size}(\mathsf{w})$.

GrB_DOMAIN_MISMATCH The domains of the vector and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_NULL_POINTER Argument indices is a NULL pointer.

Description

This variant of GrB_assign computes the result of assigning a constant scalar value – either val or s – to locations in a destination GraphBLAS vector. Either w(indices) = val or w(indices) = s is performed. If an optional binary accumulation operator ($\odot$) is provided, then either w(indices) = w(indices) $\odot$ val or w(indices) = w(indices) $\odot$ s is performed. More explicitly, if a non-opaque value val is provided:

$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{val},\ \forall\ i : 0 \leq i < \mathsf{nindices}, \quad \text{or}$$
$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{val},\ \forall\ i : 0 \leq i < \mathsf{nindices}.$$

Correspondingly, if a GrB_Scalar s is provided:

$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{s},\ \forall\ i : 0 \leq i < \mathsf{nindices}, \quad \text{or}$$
$$\mathsf{w}(\mathsf{indices}[i]) = \mathsf{w}(\mathsf{indices}[i]) \odot \mathsf{s},\ \forall\ i : 0 \leq i < \mathsf{nindices}.$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to two argument vectors are used in the GrB_assign operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)

The argument scalar, vectors, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with either $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator.

4. If accum is not GrB_NULL, then either $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal vectors, mask and index array used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\}\rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_STRUCTURE}$ is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.
   (c) If $\mathsf{desc}[\mathsf{GrB\_MASK}].\mathsf{GrB\_COMP}$ is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.
3. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GrB_Scalar version only).
4. The internal index array, $\widetilde{\boldsymbol{I}}$, is computed from argument indices as follows:
   (a) If $\mathsf{indices} = \mathsf{GrB\_ALL}$, then $\widetilde{\boldsymbol{I}}[i] = i,\ \forall\ i : 0 \leq i < \mathsf{nindices}$.
   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{indices}[i],\ \forall\ i : 0 \leq i < \mathsf{nindices}$.

The internal vector and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$
2. $0 \leq \mathsf{nindices} \leq \mathbf{size}(\widetilde{\mathbf{w}})$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the copies of the scalar, either val or $\tilde{s}$, in their destination locations relative to $\widetilde{\mathbf{w}}$.

- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows. If a non-opaque scalar val is provided:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{val}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\boldsymbol{I}}[i], \mathsf{val})\ \forall\ i,\ 0 \le i < \mathsf{nindices}\}\rangle.$$

Correspondingly, if a non-empty GrB_Scalar $\tilde{s}$ is provided (i.e., $\mathbf{size}(\tilde{s}) = 1$):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\tilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(\widetilde{\boldsymbol{I}}[i], \mathbf{val}(\tilde{s}))\ \forall\ i,\ 0 \le i < \mathsf{nindices}\}\rangle.$$

Finally, if an empty GrB_Scalar $\tilde{s}$ is provided (i.e., $\mathbf{size}(\tilde{s}) = 0$):

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\tilde{s}), \mathbf{size}(\widetilde{\mathbf{w}}), \emptyset\rangle.$$

If $\widetilde{\boldsymbol{I}}$ is empty, this operation results in an empty vector, $\widetilde{\mathbf{t}}$. Otherwise, if any value in the $\widetilde{\boldsymbol{I}}$ array is not in the range $[0,\ \mathbf{size}(\widetilde{\mathbf{w}}))$, the execution of GrB_assign ends and the index out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result vector, w, is invalid from this point forward in the sequence.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}}$ is defined as

  $$\widetilde{\mathbf{z}} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i), \forall i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The above expression defines the structure of vector $\widetilde{\mathbf{z}}$ as follows: We start with the structure of $\widetilde{\mathbf{w}}$ ($\mathbf{ind}(\widetilde{\mathbf{w}})$) and remove from it all the indices of $\widetilde{\mathbf{w}}$ that are in the set of indices being assigned ($\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}})$). Finally, we add the structure of $\widetilde{\mathbf{t}}$ ($\mathbf{ind}(\widetilde{\mathbf{t}})$).

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

  $$z_i = \widetilde{\mathbf{w}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\{\widetilde{\boldsymbol{I}}[k], \forall k\} \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  $$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in \mathbf{ind}(\widetilde{\mathbf{t}}),$$

  where the difference operator refers to set difference. We note that in this case of assigning a constant, $\{\widetilde{\boldsymbol{I}}[k], \forall k\}$ and $\mathbf{ind}(\widetilde{\mathbf{t}})$ are identical.

- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

  $$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

  $$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

  $$z_i = \widetilde{\mathbf{w}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  $$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a “write mask”.

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.7.6 assign: Constant matrix variant

Assign the same value to a specified subset of matrix elements. With the use of GrB_ALL, the entire destination matrix can be filled with the constant.

C Syntax

```
GrB_Info GrB_assign(GrB_Matrix            C,
                    const GrB_Matrix      Mask,
                    const GrB_BinaryOp    accum,
                    <type>                val,
                    const GrB_Index      *row_indices,
                    GrB_Index             nrows,
                    const GrB_Index      *col_indices,
                    GrB_Index             ncols,
                    const GrB_Descriptor  desc);

GrB_Info GrB_assign(GrB_Matrix            C,
                    const GrB_Matrix      Mask,
                    const GrB_BinaryOp    accum,
                    const GrB_Scalar      s,
                    const GrB_Index      *row_indices,
                    GrB_Index             nrows,
```

```
                                const GrB_Index      *col_indices,
                                GrB_Index             ncols,
                                const GrB_Descriptor  desc);
```

Parameters

- **C** (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the assign operation. On output, the matrix holds the results of the operation.

- **Mask** (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

- **accum** (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- **val** (IN) Scalar value to assign to (a subset of) C.

- **s** (IN) Scalar value to assign to (a subset of) C.

- **row_indices** (IN) Pointer to the ordered set (array) of indices corresponding to the rows of C that are assigned. If all rows of C are to be assigned in order from 0 to $\mathsf{nrows} - 1$, then GrB_ALL can be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. Unlike other variants, if there are duplicated values in this array the result is still defined.

- **nrows** (IN) The number of values in row_indices array. Must be in the range: $[0, \mathbf{nrows}(\mathsf{C})]$. If nrows is zero, the operation becomes a NO-OP.

- **col_indices** (IN) Pointer to the ordered set (array) of indices corresponding to the columns of C that are assigned. If all columns of C are to be assigned in order from 0 to $\mathsf{ncols} - 1$, then GrB_ALL should be specified. Regardless of execution mode and return value, this array may be manipulated by the caller after this operation returns without affecting any deferred computations for this operation. Unlike other variants, if there are duplicated values in this array the result is still defined.

- **ncols** (IN) The number of values in col_indices array. Must be in the range: $[0, \mathbf{ncols}(\mathsf{C})]$. If ncols is zero, the operation becomes a NO-OP.

- **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.

Return Values

- **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.
- **GrB_PANIC** Unknown internal error.
- **GrB_INVALID_OBJECT** This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
- **GrB_OUT_OF_MEMORY** Not enough memory available for the operation.
- **GrB_UNINITIALIZED_OBJECT** One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).
- **GrB_INDEX_OUT_OF_BOUNDS** A value in row_indices is greater than or equal to **nrows**(C), or a value in col_indices is greater than or equal to **ncols**(C). In non-blocking mode, this can be reported as an execution error.
- **GrB_DIMENSION_MISMATCH** Mask and C dimensions are incompatible, nrows is not less than **nrows**(C), or ncols is not less than **ncols**(C).
- **GrB_DOMAIN_MISMATCH** The domains of the matrix and scalar are incompatible with each other or the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
- **GrB_NULL_POINTER** Either argument row_indices is a NULL pointer, argument col_indices is a NULL pointer, or both.

Description

This variant of GrB_assign computes the result of assigning a constant scalar value – either val or s – to locations in a destination GraphBLAS matrix: Either C(row_indices, col_indices) = val or $\mathsf{C}(\mathsf{row\_indices}, \mathsf{col\_indices}) = \mathsf{s}$ is performed. If an optional binary accumulation operator ($\odot$) is provided, then either $\mathsf{C}(\mathsf{row\_indices}, \mathsf{col\_indices}) = \mathsf{C}(\mathsf{row\_indices}, \mathsf{col\_indices}) \odot \mathsf{val}$ or $\mathsf{C}(\mathsf{row\_indices}, \mathsf{col\_indices}) = \mathsf{C}(\mathsf{row\_indices}, \mathsf{col\_indices}) \odot \mathsf{s}$ is performed. More explicitly, if a non-opaque value val is provided:

$$\begin{aligned}
\mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) &= \mathsf{val}, \text{ or} \\
\mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) &= \mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) \odot \mathsf{val} \\
& \qquad\qquad \forall\ (i,j)\ :\ 0 \le i < \mathsf{nrows},\ 0 \le j < \mathsf{ncols}
\end{aligned}$$

Correspondingly, if a GrB_Scalar s is provided:

$$\begin{aligned}
\mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) &= \mathsf{s}, \text{ or} \\
\mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) &= \mathsf{C}(\mathsf{row_indices}[i], \mathsf{col_indices}[j]) \odot \mathsf{s} \\
& \qquad\qquad \forall\ (i,j)\ :\ 0 \le i < \mathsf{nrows},\ 0 \le j < \mathsf{ncols}
\end{aligned}$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to two argument matrices are used in the GrB_assign operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij})\} \rangle$

2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i,j,M_{ij})\} \rangle$ (optional)

The argument scalar, matrices, and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.

2. $\mathbf{D}(\mathsf{C})$ must be compatible with either $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{val})$, depending on the signature of the method.

3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator.

4. If accum is not GrB_NULL, then either $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_assign ends and the domain mismatch error listed above is returned.

From the arguments, the internal matrices, index arrays, and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask $\widetilde{\mathbf{M}}$ is computed from argument Mask as follows:
   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GrB_Scalar version only).
4. The internal row index array, $\widetilde{\boldsymbol{I}}$, is computed from argument row_indices as follows:
   (a) If $\mathsf{row\_indices} = \mathsf{GrB\_ALL}$, then $\widetilde{\boldsymbol{I}}[i] = i, \forall i : 0 \leq i < \mathsf{nrows}$.
   (b) Otherwise, $\widetilde{\boldsymbol{I}}[i] = \mathsf{row\_indices}[i], \forall i : 0 \leq i < \mathsf{nrows}$.
5. The internal column index array, $\widetilde{\boldsymbol{J}}$, is computed from argument col_indices as follows:
   (a) If $\mathsf{col\_indices} = \mathsf{GrB\_ALL}$, then $\widetilde{\boldsymbol{J}}[j] = j, \forall j : 0 \leq j < \mathsf{ncols}$.
   (b) Otherwise, $\widetilde{\boldsymbol{J}}[j] = \mathsf{col\_indices}[j], \forall j : 0 \leq j < \mathsf{ncols}$.

The internal matrix and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $0 \leq \mathsf{nrows} \leq \mathbf{nrows}(\widetilde{\mathbf{C}})$.
4. $0 \leq \mathsf{ncols} \leq \mathbf{ncols}(\widetilde{\mathbf{C}})$.

If any compatibility rule above is violated, execution of GrB_assign ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the assign and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the copies of the scalar, either val or $\tilde{s}$, in their destination locations relative to $\widetilde{\mathbf{C}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows. If a non-opaque scalar val is provided:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{val}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}),\\ \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathsf{val})\ \forall\ (i,j),\ 0 \le i < \mathsf{nrows},\ 0 \le j < \mathsf{ncols}\}\rangle.$$

Correspondingly, if a non-empty GrB_Scalar $\tilde{s}$ is provided (i.e., $\mathbf{size}(\tilde{s}) = 1$):

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\tilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}),\\ \{(\widetilde{\boldsymbol{I}}[i], \widetilde{\boldsymbol{J}}[j], \mathbf{val}(\tilde{s}))\ \forall\ (i,j),\ 0 \le i < \mathsf{nrows},\ 0 \le j < \mathsf{ncols}\}\rangle.$$

Finally, if an empty GrB_Scalar $\tilde{s}$ is provided (i.e., $\mathbf{size}(\tilde{s}) = 0$):

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\tilde{s}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \emptyset\rangle.$$

If either $\widetilde{\boldsymbol{I}}$ or $\widetilde{\boldsymbol{J}}$ is empty, this operation results in an empty matrix, $\widetilde{\mathbf{T}}$. Otherwise, if any value in the $\widetilde{\boldsymbol{I}}$ array is not in the range $[0,\ \mathbf{nrows}(\widetilde{\mathbf{C}}))$ or any value in the $\widetilde{\boldsymbol{J}}$ array is not in the range $[0,\ \mathbf{ncols}(\widetilde{\mathbf{C}}))$, the execution of GrB_assign ends and the index out-of-bounds error listed above is generated. In GrB_NONBLOCKING mode, the error can be deferred until a sequence-terminating GrB_wait() is called. Regardless, the result matrix C is invalid from this point forward in the sequence.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}}$ is defined as

  $$\begin{aligned}\widetilde{\mathbf{Z}} \;=\; & \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}),\\ & \{(i,j,Z_{ij}) \forall (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.\end{aligned}$$

  The above expression defines the structure of matrix $\widetilde{\mathbf{Z}}$ as follows: We start with the structure of $\widetilde{\mathbf{C}}$ ($\mathbf{ind}(\widetilde{\mathbf{C}})$) and remove from it all the indices of $\widetilde{\mathbf{C}}$ that are in the set of indices being assigned ($\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}})$). Finally, we add the structure of $\widetilde{\mathbf{T}}$ ($\mathbf{ind}(\widetilde{\mathbf{T}})$).

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\} \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  $$Z_{ij} = \widetilde{\mathbf{T}}(i,j),\ \text{if } (i,j) \in \mathbf{ind}(\widetilde{\mathbf{T}}),$$

  where the difference operator refers to set difference. We note that, in this particular case of assigning a constant to a matrix, the sets $\{(\widetilde{\boldsymbol{I}}[k], \widetilde{\boldsymbol{J}}[l]), \forall k,l\}$ and $\mathbf{ind}(\widetilde{\mathbf{T}})$ are identical.

- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i,j,Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a “write mask”.

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8 apply: Apply a function to the elements of an object

Computes the transformation of the values of the elements of a vector or a matrix using a unary function, or a binary function where one argument is bound to a scalar.

4.3.8.1 apply: Vector variant

Computes the transformation of the values of the elements of a vector using a unary function.

C Syntax

```
GrB_Info GrB_apply(GrB_Vector             w,
                   const GrB_Vector       mask,
                   const GrB_BinaryOp     accum,
                   const GrB_UnaryOp      op,
                   const GrB_Vector       u,
                   const GrB_Descriptor   desc);
```

Parameters

- **w** (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.
- **mask** (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.
- **accum** (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.
- **op** (IN) A unary operator applied to each element of input vector u.
- **u** (IN) The GraphBLAS vector to which the unary function is applied.
- **desc** (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

- **GrB_SUCCESS** In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully.

Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or unary function, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_apply computes the result of applying a unary function to the elements of a GraphBLAS vector: $\mathsf{w} = f(\mathsf{u})$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{w} = \mathsf{w} \odot f(\mathsf{u})$.

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB_apply operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument vectors, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.

2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator.

3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

4. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in}(\mathsf{op})$.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.

2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:

   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\}\rangle$.

   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,

      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,

      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.

   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.

3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$

2. $\mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{size}(\widetilde{\mathbf{w}})$.

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the unary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle,$$

where $f = \mathbf{f}(\mathsf{op})$.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

  $$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

  $$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

  $$z_i = \widetilde{\mathbf{w}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  $$z_i = \widetilde{\mathbf{t}}(i),\ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

  $$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

  $$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8.2 apply: Matrix variant

Computes the transformation of the values of the elements of a matrix using a unary function.

C Syntax

```
GrB_Info GrB_apply(GrB_Matrix             C,
                   const GrB_Matrix       Mask,
                   const GrB_BinaryOp     accum,
                   const GrB_UnaryOp      op,
                   const GrB_Matrix       A,
                   const GrB_Descriptor   desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) A unary operator applied to each element of input matrix A.

A (IN) The GraphBLAS matrix to which the unary function is applied.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_DIMENSION_MISMATCH Mask and C dimensions are incompatible, nrows $\neq$ **nrows**(C), or ncols $\neq$ **ncols**(C).

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or unary function, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_apply computes the result of applying a unary function to the elements of a GraphBLAS matrix: $\mathsf{C} = f(\mathsf{A})$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot f(\mathsf{A})$.

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_apply operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)

3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\}\rangle$

The argument matrices, unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in}(\mathsf{op})$ of the unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i, j), \forall i, j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If Mask $\neq$ GrB_NULL,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
      $\{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i, j) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A.

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.

4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the unary operator to the input matrix $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i,j))) \ \forall \ (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})\} \rangle,$$

where $f = \mathbf{f}(\mathsf{op})$.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

  $$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  $$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

  $$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8.3 apply: Vector-BinaryOp variants

Computes the transformation of the values of the stored elements of a vector using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the vector are passed as the second argument. In the *bind-second* variant, the elements of the vector are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

C Syntax

```
// bind-first + scalar value
GrB_Info GrB_apply(GrB_Vector             w,
                   const GrB_Vector       mask,
                   const GrB_BinaryOp     accum,
                   const GrB_BinaryOp     op,
                   <type>                 val,
                   const GrB_Vector       u,
                   const GrB_Descriptor   desc);

// bind-first + GraphBLAS scalar
GrB_Info GrB_apply(GrB_Vector             w,
                   const GrB_Vector       mask,
                   const GrB_BinaryOp     accum,
                   const GrB_BinaryOp     op,
                   const GrB_Scalar       s,
                   const GrB_Vector       u,
                   const GrB_Descriptor   desc);

// bind-second + scalar value
GrB_Info GrB_apply(GrB_Vector             w,
                   const GrB_Vector       mask,
```

```
                          const GrB_BinaryOp     accum,
                          const GrB_BinaryOp     op,
                          const GrB_Vector       u,
                          <type>                 val,
                          const GrB_Descriptor   desc);

// bind-second + GraphBLAS scalar
GrB_Info GrB_apply(GrB_Vector                    w,
                          const GrB_Vector       mask,
                          const GrB_BinaryOp     accum,
                          const GrB_BinaryOp     op,
                          const GrB_Vector       u,
                          const GrB_Scalar       s,
                          const GrB_Descriptor   desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) A binary operator applied to each element of input vector, u, and the scalar value, val.

u (IN) The GraphBLAS vector whose elements are passed to the binary operator as the right-hand (second) argument in the *bind-first* variant, or the left-hand (first) argument in the *bind-second* variant.

val (IN) Scalar value that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant.

s (IN) A GraphBLAS scalar that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_EMPTY_OBJECT The GrB_Scalar s used in the call is empty (**nvals**(s) = 0) and therefore a value cannot be passed to the binary operator.

Description

This variant of GrB_apply computes the result of applying a binary operator to the elements of a GraphBLAS vector each composed with a scalar constant, either val or s:

$$\text{bind-first:} \quad \mathsf{w} = f(\mathsf{val}, \mathsf{u}) \text{ or } \mathsf{w} = f(\mathsf{s}, \mathsf{u})$$

$$\text{bind-second:} \quad \mathsf{w} = f(\mathsf{u}, \mathsf{val}) \text{ or } \mathsf{w} = f(\mathsf{u}, \mathsf{s}),$$

or if an optional binary accumulation operator ($\odot$) is provided:

$$\text{bind-first:} \quad \mathsf{w} = \mathsf{w} \odot f(\mathsf{val}, \mathsf{u}) \text{ or } \mathsf{w} = \mathsf{w} \odot f(\mathsf{s}, \mathsf{u})$$

$$\text{bind-second:} \quad \mathsf{w} = \mathsf{w} \odot f(\mathsf{u}, \mathsf{val}) \text{ or } \mathsf{w} = \mathsf{w} \odot f(\mathsf{u}, \mathsf{s}).$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB_apply operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument scalar, vectors, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
5. If bind-first:
   (a) $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.
   (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
   (c) If the GrB_Scalar s is provided, then $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.

6. If bind-second:
   (a) $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
   (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.
   (c) If the GrB_Scalar s is provided, then $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:
   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.
   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.
3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.
4. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$
2. $\mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{size}(\widetilde{\mathbf{w}})$.

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar $\tilde{s}$ is provided ($\mathbf{nvals}(\tilde{s}) = 0$), the method returns with code GrB_EMPTY_OBJECT. If a non-empty GrB_Scalar, $\tilde{s}$, is provided (i.e., $\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable val with the same domain as $\tilde{s}$ and set $\mathsf{val} = \mathbf{val}(\tilde{s})$.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the binary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as one of the following:

$$\text{bind-first:} \quad \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\mathsf{val}, \widetilde{\mathbf{u}}(i))) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle,$$

$$\text{bind-second:} \quad \widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f(\widetilde{\mathbf{u}}(i), \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle,$$

where $f = \mathbf{f}(\mathsf{op})$.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \ \text{if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8.4 apply: Matrix-BinaryOp variants

Computes the transformation of the values of the stored elements of a matrix using a binary operator and a scalar value. In the *bind-first* variant, the specified scalar value is passed as the first argument to the binary operator and stored elements of the matrix are passed as the second argument. In the *bind-second* variant, the elements of the matrix are passed as the first argument and the specified scalar value is passed as the second argument. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

C Syntax

```
        // bind-first + scalar value
        GrB_Info GrB_apply(GrB_Matrix             C,
                           const GrB_Matrix       Mask,
                           const GrB_BinaryOp     accum,
                           const GrB_BinaryOp     op,
                           <type>                 val,
                           const GrB_Matrix       A,
                           const GrB_Descriptor   desc);

        // bind-first + GraphBLAS scalar
        GrB_Info GrB_apply(GrB_Matrix             C,
                           const GrB_Matrix       Mask,
                           const GrB_BinaryOp     accum,
                           const GrB_BinaryOp     op,
                           const GrB_Scalar       s,
                           const GrB_Matrix       A,
                           const GrB_Descriptor   desc);

        // bind-second + scalar value
        GrB_Info GrB_apply(GrB_Matrix             C,
                           const GrB_Matrix       Mask,
                           const GrB_BinaryOp     accum,
                           const GrB_BinaryOp     op,
                           const GrB_Matrix       A,
                           <type>                 val,
                           const GrB_Descriptor   desc);

        // bind-second + GraphBLAS scalar
        GrB_Info GrB_apply(GrB_Matrix             C,
                           const GrB_Matrix       Mask,
                           const GrB_BinaryOp     accum,
                           const GrB_BinaryOp     op,
                           const GrB_Matrix       A,
```

```
                                  const GrB_Scalar      s,
                                  const GrB_Descriptor  desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) A binary operator applied to each element of input matrix, A, with the element of the input matrix used as the left-hand argument, and the scalar value, val, used as the right-hand argument.

A (IN) The GraphBLAS matrix whose elements are passed to the binary operator as the right-hand (second) argument in the *bind-first* variant, or the left-hand (first) argument in the *bind-second* variant.

val (IN) Scalar value that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant.

s (IN) GraphBLAS scalar value that is passed to the binary operator as the left-hand (first) argument in the *bind-first* variant, or the right-hand (second) argument in the *bind-second* variant. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation (*bind-second* variant only).
A	GrB_INP1	GrB_TRAN	Use transpose of A for the operation (*bind-first* variant only).

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_INDEX_OUT_OF_BOUNDS A value in row_indices is greater than or equal to **nrows**(A), or a value in col_indices is greater than or equal to **ncols**(A). In non-blocking mode, this can be reported as an execution error.

GrB_DIMENSION_MISMATCH Mask and C dimensions are incompatible, nrows $\neq$ **nrows**(C), or ncols $\neq$ **ncols**(C).

GrB_DOMAIN_MISMATCH The domains of the various matrices and scalar are incompatible with the corresponding domains of the binary operator or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_EMPTY_OBJECT The GrB_Scalar s used in the call is empty (**nvals**(s) $= 0$) and therefore a value cannot be passed to the binary operator.

Description

This variant of GrB_apply computes the result of applying a binary operator to the elements of a GraphBLAS matrix each composed with a scalar constant, val or s:

$$\text{bind-first:} \quad \mathsf{C} = f(\mathsf{val}, \mathsf{A}) \text{ or } \mathsf{C} = f(\mathsf{s}, \mathsf{A})$$

$$\text{bind-second:} \quad \mathsf{C} = f(\mathsf{A}, \mathsf{val}) \text{ or } \mathsf{C} = f(\mathsf{A}, \mathsf{s}),$$

or if an optional binary accumulation operator ($\odot$) is provided:

$$\text{bind-first:} \quad \mathsf{C} = \mathsf{C} \odot f(\mathsf{val}, \mathsf{A}) \text{ or } \mathsf{C} = \mathsf{C} \odot f(\mathsf{s}, \mathsf{A})$$

$$\text{bind-second:} \quad \mathsf{C} = \mathsf{C} \odot f(\mathsf{A}, \mathsf{val}) \text{ or } \mathsf{C} = \mathsf{C} \odot f(\mathsf{A}, \mathsf{s}).$$

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_apply operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$

The argument scalar, matrices, binary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the binary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.
5. If bind-first:
   (a) $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

(b) If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.

(c) If the GrB_Scalar s is provided, then $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.

6. If bind-second:

   (a) $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the binary operator.

   (b) If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

   (c) If the GrB_Scalar s is provided, then $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the binary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.

2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:

   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.

   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,

      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,

      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.

   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.

3. Matrix $\widetilde{\mathbf{A}}$ is computed from argument A as follows:

$$\text{bind-first:} \quad \widetilde{\mathbf{A}} \leftarrow \mathsf{desc[GrB\_INP1].GrB\_TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$$
$$\text{bind-second:} \quad \widetilde{\mathbf{A}} \leftarrow \mathsf{desc[GrB\_INP0].GrB\_TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$$

4. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.

2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.

3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.

4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar $\tilde{s}$ is provided ($\mathbf{nvals}(\tilde{s}) = 0$), the method returns with code GrB_EMPTY_OBJECT. If a non-empty GrB_Scalar, $\tilde{s}$, is provided (i.e., $\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable val with the same domain as $\tilde{s}$ and set $\mathsf{val} = \mathbf{val}(\tilde{s})$.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the binary operator to the input matrix $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as one of the following:

$$\text{bind-first:} \quad \widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\mathsf{val}, \widetilde{\mathbf{A}}(i,j))) \ \forall \ (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})\} \rangle,$$

$$\text{bind-second:} \quad \widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f(\widetilde{\mathbf{A}}(i,j), \mathsf{val})) \ \forall \ (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})\} \rangle,$$

where $f = \mathbf{f}(\mathsf{op})$.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\} \rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8.5 apply: Vector index unary operator variant

Computes the transformation of the values of the stored elements of a vector using an index unary operator that is a function of the stored value, its location indices, and an user provided scalar value. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

C Syntax

```
GrB_Info GrB_apply(GrB_Vector             w,
                   const GrB_Vector       mask,
                   const GrB_BinaryOp     accum,
                   const GrB_IndexUnaryOp op,
                   const GrB_Vector       u,
                   <type>                 val,
                   const GrB_Descriptor   desc);

GrB_Info GrB_apply(GrB_Vector             w,
                   const GrB_Vector       mask,
                   const GrB_BinaryOp     accum,
                   const GrB_IndexUnaryOp op,
                   const GrB_Vector       u,
                   const GrB_Scalar       s,
                   const GrB_Descriptor   desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the apply operation. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input vector, u. It is a function of the stored element's value, its location index, and a user supplied scalar value (either s or val).

u (IN) The GraphBLAS vector whose elements are passed to the index unary operator.

val (IN) An additional scalar value that is passed to the index unary operator.

s (IN) An additional GraphBLAS scalar that is passed to the index unary operator. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).

GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

GrB_EMPTY_OBJECT The GrB_Scalar s used in the call is empty ($\mathbf{nvals}(\mathsf{s}) = 0$) and therefore a value cannot be passed to the index unary operator.

Description

This variant of GrB_apply computes the result of applying an index unary operator to the elements of a GraphBLAS vector each composed with the element's index and a scalar constant, val or s:

$$\mathsf{w} = f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val}) \text{ or } \mathsf{w} = f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}),$$

or if an optional binary accumulation operator ($\odot$) is provided:

$$\mathsf{w} = \mathsf{w} \odot f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val}) \text{ or } \mathsf{w} = \mathsf{w} \odot f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s}).$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB_apply operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)

3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.

2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator.

3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

4. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.

5. If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

6. If the GrB_Scalar s is provided, then $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.

2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:

   (a) If $\mathsf{mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i, \ \forall \ i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.

   (b) If $\mathsf{mask} \neq \mathsf{GrB\_NULL}$,

      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,

      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.

   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.

3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

4. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GraphBLAS scalar case).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$

2. $\mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{size}(\widetilde{\mathbf{w}})$.

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar $\tilde{s}$ is provided ($\mathbf{nvals}(\tilde{s}) = 0$), the method returns with code GrB_EMPTY_OBJECT. If a non-empty GrB_Scalar, $\tilde{s}$, is provided ($\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable val with the same domain as $\tilde{s}$ and set $\mathsf{val} = \mathbf{val}(\tilde{s})$.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the index unary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, f_i(\widetilde{\mathbf{u}}(i), [i], 0, \mathsf{val})) \forall i \in \mathbf{ind}(\widetilde{\mathbf{u}})\}\rangle,$$

where $f_i = \mathbf{f}(\mathsf{op})$.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

  $$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

  $$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

  $$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  $$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.8.6 apply: Matrix index unary operator variant

Computes the transformation of the values of the stored elements of a matrix using an index unary operator that is a function of the stored value, its location indices, and an user provided scalar value. The scalar can be passed either as a non-opaque variable or as a GrB_Scalar object.

C Syntax

```
GrB_Info GrB_apply(GrB_Matrix              C,
                   const GrB_Matrix        Mask,
                   const GrB_BinaryOp      accum,
                   const GrB_IndexUnaryOp  op,
                   const GrB_Matrix        A,
                   <type>                  val,
                   const GrB_Descriptor    desc);

GrB_Info GrB_apply(GrB_Matrix              C,
                   const GrB_Matrix        Mask,
                   const GrB_BinaryOp      accum,
                   const GrB_IndexUnaryOp  op,
                   const GrB_Matrix        A,
                   const GrB_Scalar        s,
                   const GrB_Descriptor    desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the apply operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).

A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.

val (IN) An additional scalar value that is passed to the index unary operator.

s (IN) An additional GraphBLAS scalar that is passed to the index unary operator.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused

	by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to new (or another constructor).
GrB_DIMENSION_MISMATCH	mask, w and/or u dimensions are incompatible.
GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_EMPTY_OBJECT	The GrB_Scalar s used in the call is empty ($\mathbf{nvals}(\mathsf{s}) = 0$) and therefore a value cannot be passed to the index unary operator.

Description

This variant of GrB_apply computes the result of applying a index unary operator to the elements of a GraphBLAS matrix each composed with the elements row and column indices, and a scalar constant, val or s:

$$\mathsf{C} = f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val}) \text{ or } \mathsf{C} = f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{s}),$$

or if an optional binary accumulation operator ($\odot$) is provided:

$$\mathsf{C} = \mathsf{C} \odot f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{val}) \text{ or } \mathsf{C} = \mathsf{C} \odot f_i(\mathsf{A}, \mathbf{row}(\mathbf{ind}(\mathsf{A})), \mathbf{col}(\mathbf{ind}(\mathsf{A})), \mathsf{s}).$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_apply operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)

3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$

The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.
5. If the non-opaque scalar val is provided, then $\mathbf{D}(\mathsf{val})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.
6. If the GrB_Scalar s is provided, then $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_apply ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i, j), \forall i, j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\} \rangle$.
   (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask})\} \rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$ $\{(i, j) : (i, j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i, j) = \mathsf{true}\} \rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg \widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}}$ is computed from argument A as follows:

$$\widetilde{\mathbf{A}} \leftarrow \mathsf{desc[GrB\_INP0].GrB\_TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$$

4. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GraphBLAS scalar case).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.
4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of GrB_apply ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar $\tilde{s}$ is provided ($\mathbf{nvals}(\tilde{s}) = 0$), the method returns with code GrB_EMPTY_OBJECT. If a non-empty GrB_Scalar, $\tilde{s}$, is provided (i.e., $\mathbf{nvals}(\tilde{s}) = 1$), we then create an internal variable val with the same domain as $\tilde{s}$ and set $\mathsf{val} = \mathbf{val}(\tilde{s})$.

We are now ready to carry out the apply and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the index unary operator to the input matrix $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, f_i(\widetilde{\mathbf{A}}(i,j), i, j, \mathsf{val}))\ \forall\ (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})\}\rangle,$$

where $f_i = \mathbf{f}(\mathsf{op})$.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

  $$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

  $$Z_{ij} = \widetilde{\mathbf{C}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  $$Z_{ij} = \widetilde{\mathbf{T}}(i,j),\ \text{if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.9 select:

Apply a select operator to the stored elements of an object to determine whether or not to keep them.

4.3.9.1 select: Vector variant

Apply a select operator (an index unary operator) to the elements of a vector.

C Syntax

```
        // scalar value variant
        GrB_Info GrB_select(GrB_Vector             w,
                            const GrB_Vector       mask,
                            const GrB_BinaryOp     accum,
                            const GrB_IndexUnaryOp  op,
                            const GrB_Vector       u,
                            <type>                 val,
                            const GrB_Descriptor   desc);

        // GraphBLAS scalar variant
        GrB_Info GrB_select(GrB_Vector             w,
                            const GrB_Vector       mask,
```

```
                                    const GrB_BinaryOp      accum,
                                    const GrB_IndexUnaryOp  op,
                                    const GrB_Vector        u,
                                    const GrB_Scalar        s,
                                    const GrB_Descriptor    desc);
```

Parameters

w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the select operation. On output, this vector holds the results of the operation.

mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\text{GrB\_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input vector, u. It is a function of the stored element's value, its location index, and a user supplied scalar value (either s or val).

u (IN) The GraphBLAS vector whose elements are passed to the index unary operator.

val (IN) An additional scalar value that is passed to the index unary operator.

s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.
GrB_DIMENSION_MISMATCH	mask, w and/or u dimensions are incompatible.
GrB_DOMAIN_MISMATCH	The domains of the various vectors are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_EMPTY_OBJECT	The GrB_Scalar s used in the call is empty ($\mathbf{nvals}(\mathsf{s}) = 0$) and therefore a value cannot be passed to the index unary operator.

Description

This variant of GrB_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS vector. The operator takes, as input, the value of each stored element, along with the element's index and a scalar constant – either val or s. The corresponding element of the input vector is selected (kept) if the function evaluates to true when cast to bool. This acts like a functional mask on the input vector as follows:

$$\mathsf{w} = \mathsf{u}\langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val})\rangle,$$

$$\mathsf{w} = \mathsf{w} \odot \mathsf{u}\langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{val})\rangle.$$

Correspondingly, if a GrB_Scalar, s, is provided:

$$\mathsf{w} = \mathsf{u}\langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s})\rangle,$$

$$\mathsf{w} = \mathsf{w} \odot \mathsf{u}\langle f_i(\mathsf{u}, \mathbf{ind}(\mathsf{u}), 0, \mathsf{s})\rangle.$$

Logically, this operation occurs in three steps:

Setup The internal vectors and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to three argument vectors are used in this GrB_select operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The argument scalar, vectors, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}(\mathsf{u})$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
5. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.
6. $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_select ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.
2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:

(a) If mask $=$ GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\}\rangle$.

(b) If mask $\neq$ GrB_NULL,

i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\}\rangle$,

ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\}\rangle$.

(c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg\widetilde{\mathbf{m}}$.

3. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

4. Scalar $\widetilde{\mathsf{s}} \leftarrow \mathsf{s}$ (GrB_Scalar version only).

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$

2. $\mathbf{size}(\widetilde{\mathbf{u}}) = \mathbf{size}(\widetilde{\mathbf{w}})$.

If any compatibility rule above is violated, execution of GrB_select ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar $\widetilde{\mathsf{s}}$ is provided (i.e., $\mathbf{nvals}(\widetilde{\mathsf{s}}) = 0$), the method returns with code GrB_EMPTY_OBJECT. If a non-empty GrB_Scalar, $\widetilde{\mathsf{s}}$, is provided (i.e., $\mathbf{nvals}(\widetilde{\mathsf{s}}) = 1$), we then create an internal variable val with the same domain as $\widetilde{\mathsf{s}}$ and set val $= \mathbf{val}(\widetilde{\mathsf{s}})$.

We are now ready to carry out the select and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from applying the index unary operator to the input vector $\widetilde{\mathbf{u}}$.
- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\widetilde{\mathbf{u}}), \{(i, \widetilde{\mathbf{u}}(i), : i \in \mathbf{ind}(\widetilde{\mathbf{u}}) \wedge (\mathsf{bool}) f_i(\widetilde{\mathbf{u}}(i), i, 0, \mathsf{val}) = \mathsf{true}\}\rangle,$$

where $f_i = \mathbf{f}(\mathsf{op})$.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum $=$ GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i)\ \forall\ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.9.2 select: Matrix variant

Apply a select operator (an index unary operator) to the elements of a matrix.

C Syntax

```
// scalar value variant
GrB_Info GrB_select(GrB_Matrix              C,
                    const GrB_Matrix        Mask,
                    const GrB_BinaryOp      accum,
                    const GrB_IndexUnaryOp  op,
                    const GrB_Matrix        A,
                    <type>                  val,
                    const GrB_Descriptor    desc);
```

```
// GraphBLAS scalar variant
GrB_Info GrB_select(GrB_Matrix             C,
                    const GrB_Matrix        Mask,
                    const GrB_BinaryOp      accum,
                    const GrB_IndexUnaryOp  op,
                    const GrB_Matrix        A,
                    const GrB_Scalar        s,
                    const GrB_Descriptor    desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the select operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) An index unary operator, $F_i = \langle D_{out}, D_{in_1}, \mathbf{D}(\mathsf{GrB\_Index}), D_{in_2}, f_i \rangle$, applied to each element stored in the input matrix, A. It is a function of the stored element's value, its row and column indices, and a user supplied scalar value (either s or val).

A (IN) The GraphBLAS matrix whose elements are passed to the index unary operator.

val (IN) An additional scalar value that is passed to the index unary operator.

s (IN) An GraphBLAS scalar that is passed to the index unary operator. It must not be empty.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS	In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output mattrix C is ready to be used in the next method of the sequence.
GrB_PANIC	Unknown internal error.
GrB_INVALID_OBJECT	This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.
GrB_OUT_OF_MEMORY	Not enough memory available for operation.
GrB_UNINITIALIZED_OBJECT	One or more of the GraphBLAS objects has not been initialized by a call to one of its constructors.
GrB_DIMENSION_MISMATCH	Mask, C and/or A dimensions are incompatible.
GrB_DOMAIN_MISMATCH	The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator or index unary operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).
GrB_EMPTY_OBJECT	The GrB_Scalar s used in the call is empty (**nvals**(s) = 0) and therefore a value cannot be passed to the index unary operator.

Description

This variant of GrB_select computes the result of applying a index unary operator to select the elements of the input GraphBLAS matrix. The operator takes, as input, the value of each stored element, along with the element's row and column indices and a scalar constant – from either val or s. The corresponding element of the input matrix is selected (kept) if the function evaluates to true when cast to bool. This acts like a functional mask on the input matrix as follows when specifying a transparent scalar value:

$$C = A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val})\rangle, \text{ or}$$

$$C = C \odot A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), \mathsf{val})\rangle.$$

Correspondingly, if a GrB_Scalar, s, is provided:

$$C = A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s)\rangle, \text{ or}$$

$$C = C \odot A\langle f_i(A, \mathbf{row}(\mathbf{ind}(A)), \mathbf{col}(\mathbf{ind}(A)), s)\rangle.$$

Where the **row** and **col** functions extract the row and column indices from a list of two-dimensional indices, respectively.

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three argument matrices are used in the GrB_select operation:

1. $C = \langle \mathbf{D}(C), \mathbf{nrows}(C), \mathbf{ncols}(C), \mathbf{L}(C) = \{(i, j, C_{ij})\}\rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\}\rangle$ (optional)
3. $A = \langle \mathbf{D}(A), \mathbf{nrows}(A), \mathbf{ncols}(A), \mathbf{L}(A) = \{(i, j, A_{ij})\}\rangle$

The argument scalar, matrices, index unary operator and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(C)$ must be compatible with $\mathbf{D}(A)$.
3. If accum is not GrB_NULL, then $\mathbf{D}(C)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}_{out}(\mathsf{op})$ of the index unary operator must be from one of the pre-defined types of Table 3.2; i.e., castable to bool.
5. $\mathbf{D}(A)$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{op})$ of the index unary operator.
6. $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(s)$, depending on the signature of the method, must be compatible with $\mathbf{D}_{in_2}(\mathsf{op})$ of the index unary operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_select ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices, mask, and index arrays used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
   (a) If Mask = GrB_NULL, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
   (b) If Mask $\neq$ GrB_NULL,
      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
      ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.
   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}}$ is computed from argument A as follows: $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A
4. Scalar $\widetilde{\mathsf{s}} \leftarrow \mathsf{s}$ (GrB_Scalar version only).

The internal matrices and mask are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.
4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of GrB_select ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

If an empty GrB_Scalar $\widetilde{\mathsf{s}}$ is provided (i.e., $\mathbf{nvals}(\widetilde{\mathsf{s}}) = 0$), the method returns with code GrB_EMPTY_OBJECT. If a non-empty GrB_Scalar, $\widetilde{\mathsf{s}}$, is provided (i.e., $\mathbf{nvals}(\widetilde{\mathsf{s}}) = 1$), we then create an internal variable val with the same domain as $\widetilde{\mathsf{s}}$ and set $\mathsf{val} = \mathbf{val}(\widetilde{\mathsf{s}})$.

We are now ready to carry out the select and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the result from applying the index unary operator to the input matrix $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix, $\widetilde{\mathbf{T}}$, is created as follows:

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \mathbf{ncols}(\widetilde{\mathbf{A}}),$$
$$\{(i, j, \widetilde{\mathbf{A}}(i, j) : i, j \in \mathbf{ind}(\widetilde{\mathbf{A}}) \wedge (\mathsf{bool}) f_i(\widetilde{\mathbf{A}}(i, j), i, j, \mathsf{val}) = \mathsf{true}\}\rangle,$$

where $f_i = \mathbf{f}(\mathsf{op})$.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i, j) \odot \widetilde{\mathbf{T}}(i, j), \text{ if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i, j), \text{ if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i, j), \text{ if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.10 reduce: Perform a reduction across the elements of an object

Computes the reduction of the values of the elements of a vector or matrix.

4.3.10.1 reduce: Standard matrix to vector variant

This performs a reduction across rows of a matrix to produce a vector. If reduction down columns is desired, the input matrix should be transposed using the descriptor.

C Syntax

```
GrB_Info GrB_reduce(GrB_Vector             w,
                    const GrB_Vector       mask,
                    const GrB_BinaryOp     accum,
                    const GrB_Monoid       op,
                    const GrB_Matrix       A,
                    const GrB_Descriptor   desc);

GrB_Info GrB_reduce(GrB_Vector             w,
                    const GrB_Vector       mask,
                    const GrB_BinaryOp     accum,
                    const GrB_BinaryOp     op,
                    const GrB_Matrix       A,
                    const GrB_Descriptor   desc);
```

Parameters

- w (INOUT) An existing GraphBLAS vector. On input, the vector provides values that may be accumulated with the result of the reduction operation. On output, this vector holds the results of the operation.

- mask (IN) An optional "write" mask that controls which results from this operation are stored into the output vector w. The mask dimensions must match those of the vector w. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the mask vector must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of w), GrB_NULL should be specified.

- accum (IN) An optional binary operator used for accumulating entries into existing w entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

- op (IN) The monoid or binary operator used in the element-wise reduction operation. Depending on which type is passed, the following defines the binary operator with one domain, $F_b = \langle D, D, D, \oplus \rangle$, that is used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigodot(\mathsf{op})\rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigodot(\mathsf{op})\rangle$, the identity element of the monoid is ignored.

If op is a GrB_BinaryOp, then all its domains must be the same. Furthermore, in both cases $\bigodot(\mathsf{op})$ must be commutative and associative. Otherwise, the outcome of the operation is undefined.

A (IN) The GraphBLAS matrix on which reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
w	GrB_OUTP	GrB_REPLACE	Output vector w is cleared (all elements removed) before the result is stored in it.
mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input mask vector. The stored values are not examined.
mask	GrB_MASK	GrB_COMP	Use the complement of mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output vector w is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or dup for vector parameters).

GrB_DIMENSION_MISMATCH mask, w and/or u dimensions are incompatible.

GrB_DOMAIN_MISMATCH Either the domains of the various vectors and matrices are incompatible with the corresponding domains of the accumulation operator or reduce function, or the domains of the GraphBLAS binary operator op are not all the same, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

This variant of GrB_reduce computes the result of performing a reduction across each of the rows of an input matrix: $\mathsf{w}(i) = \bigoplus \mathsf{A}(i,:) \forall i$; or, if an optional binary accumulation operator is provided, $\mathsf{w}(i) = \mathsf{w}(i) \odot (\bigoplus \mathsf{A}(i,:)) \forall i$, where $\bigoplus = \bigodot(F_b)$ and $\odot = \bigodot(\mathsf{accum})$.

Logically, this operation occurs in three steps:

Setup The internal vector, matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output vector, possibly under control of a mask.

Up to two vector and one matrix argument are used in this GrB_reduce operation:

1. $\mathsf{w} = \langle \mathbf{D}(\mathsf{w}), \mathbf{size}(\mathsf{w}), \mathbf{L}(\mathsf{w}) = \{(i, w_i)\} \rangle$
2. $\mathsf{mask} = \langle \mathbf{D}(\mathsf{mask}), \mathbf{size}(\mathsf{mask}), \mathbf{L}(\mathsf{mask}) = \{(i, m_i)\} \rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$

The argument vector, matrix, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

1. If mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{w})$ must be compatible with the domain of the reduction binary operator, $\mathbf{D}(F_b)$.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{w})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(F_b)$, must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
4. $\mathbf{D}(\mathsf{A})$ must be compatible with the domain of the binary reduction operator, $\mathbf{D}(F_b)$.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_reduce ends and the domain mismatch error listed above is returned.

From the argument vectors, the internal vectors and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{w}} \leftarrow \mathsf{w}$.

2. One-dimensional mask, $\widetilde{\mathbf{m}}$, is computed from argument mask as follows:

   (a) If mask = GrB_NULL, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{w}), \{i,\ \forall\ i : 0 \leq i < \mathbf{size}(\mathsf{w})\} \rangle$.

   (b) If mask $\neq$ GrB_NULL,

      i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask})\} \rangle$,

      ii. Otherwise, $\widetilde{\mathbf{m}} = \langle \mathbf{size}(\mathsf{mask}), \{i : i \in \mathbf{ind}(\mathsf{mask}) \wedge (\mathsf{bool})\mathsf{mask}(i) = \mathsf{true}\} \rangle$.

   (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{m}} \leftarrow \neg \widetilde{\mathbf{m}}$.

3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A.

The internal vectors and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{size}(\widetilde{\mathbf{m}})$

2. $\mathbf{size}(\widetilde{\mathbf{w}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of **GrB_reduce** ends and the dimension mismatch error listed above is returned.

From this point forward, in **GrB_NONBLOCKING** mode, the method can optionally exit with **GrB_SUCCESS** return code and defer any computation and/or execution error codes.

We carry out the reduce and any additional associated operations. We describe this in terms of two intermediate vectors:

- $\widetilde{\mathbf{t}}$: The vector holding the result from reducing along the rows of input matrix $\widetilde{\mathbf{A}}$.

- $\widetilde{\mathbf{z}}$: The vector holding the result after application of the (optional) accumulation operator.

The intermediate vector, $\widetilde{\mathbf{t}}$, is created as follows:

$$\widetilde{\mathbf{t}} = \langle \mathbf{D}(\mathsf{op}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, t_i) : \mathbf{ind}(A(i,:)) \neq \emptyset\} \rangle.$$

The value of each of its elements is computed by

$$t_i = \bigoplus_{j \in \mathbf{ind}(\widetilde{\mathbf{A}}(i,:))} \widetilde{\mathbf{A}}(i,j),$$

where $\bigoplus = \bigodot(F_b)$.

The intermediate vector $\widetilde{\mathbf{z}}$ is created as follows, using what is called a *standard vector accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{z}} = \widetilde{\mathbf{t}}$.

- If accum is a binary operator, then $\widetilde{\mathbf{z}}$ is defined as

$$\widetilde{\mathbf{z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{size}(\widetilde{\mathbf{w}}), \{(i, z_i) \ \forall \ i \in \mathbf{ind}(\widetilde{\mathbf{w}}) \cup \mathbf{ind}(\widetilde{\mathbf{t}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{w}}$ and $\widetilde{\mathbf{t}}$.

$$z_i = \widetilde{\mathbf{w}}(i) \odot \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}})),$$

$$z_i = \widetilde{\mathbf{w}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{w}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

$$z_i = \widetilde{\mathbf{t}}(i), \text{ if } i \in (\mathbf{ind}(\widetilde{\mathbf{t}}) - (\mathbf{ind}(\widetilde{\mathbf{t}}) \cap \mathbf{ind}(\widetilde{\mathbf{w}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up vector $\widetilde{\mathbf{z}}$ are written into the final result vector w, using what is called a *standard vector mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in w on input to this operation are deleted and the content of the new output vector, w, is defined as,

$$\mathbf{L}(\mathsf{w}) = \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{z}}$ indicated by the mask are copied into the result vector, w, and elements of w that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{w}) = \{(i, w_i) : i \in (\mathbf{ind}(\mathsf{w}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{m}}))\} \cup \{(i, z_i) : i \in (\mathbf{ind}(\widetilde{\mathbf{z}}) \cap \mathbf{ind}(\widetilde{\mathbf{m}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of vector w is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.10.2 reduce: Vector-scalar variant

Reduce all stored values into a single scalar.

C Syntax

```
        // scalar value + monoid (only)
        GrB_Info GrB_reduce(<type>              *val,
                            const GrB_BinaryOp   accum,
                            const GrB_Monoid     op,
                            const GrB_Vector     u,
```

```
                           const GrB_Descriptor  desc);

    // GraphBLAS Scalar + monoid
    GrB_Info GrB_reduce(GrB_Scalar            s,
                        const GrB_BinaryOp    accum,
                        const GrB_Monoid      op,
                        const GrB_Vector      u,
                        const GrB_Descriptor  desc);

    // GraphBLAS Scalar + binary operator
    GrB_Info GrB_reduce(GrB_Scalar            s,
                        const GrB_BinaryOp    accum,
                        const GrB_BinaryOp    op,
                        const GrB_Vector      u,
                        const GrB_Descriptor  desc);
```

Parameters

val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.

accum (IN) An optional binary operator used for accumulating entries into an existing scalar (s or val) value. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The monoid ($M = \langle D, \oplus, 0\rangle$) or binary operator ($F_b = \langle D, D, D, \oplus\rangle$) used in the reduction operation. The $\oplus$ operator must be commutative and associative; otherwise, the outcome of the operation is undefined.

u (IN) The GraphBLAS vector on which reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description

Note: This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

GrB_NULL_POINTER val pointer is NULL.

GrB_DOMAIN_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

Description

This variant of GrB_reduce computes the result of performing a reduction across all of the stored elements of an input vector storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

$$\mathsf{val} \;=\; \begin{cases} \bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i), & \text{or} \\ \mathsf{val} \;\odot\; \left[\bigoplus_{i \in \mathbf{ind}(\mathsf{u})} \mathsf{u}(i)\right], & \text{if the the optional accumulator is specified.} \end{cases}$$

where $\bigoplus = \bigodot(\mathsf{op})$ and $\odot = \bigodot(\mathsf{accum})$.

Logically, this operation occurs in three steps:

Setup The internal vector used in the computation is formed and its domain is tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One vector argument is used in this GrB_reduce operation:

1. $\mathsf{u} = \langle \mathbf{D}(\mathsf{u}), \mathbf{size}(\mathsf{u}), \mathbf{L}(\mathsf{u}) = \{(i, u_i)\} \rangle$

The output scalar, argument vector, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

1. If accum is GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).

2. If accum is not $\mathsf{GrB\_NULL}$, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator, and $\mathbf{D}(\mathsf{op})$ from M (or $\mathbf{D}_{out}(\mathsf{op})$ from F_b) must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

3. $\mathbf{D}(\mathsf{u})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, u, is checked. If there are no stored values in u, then one of the following occurs depending on the output variant:

$$\mathbf{L}(\mathsf{s}) = \begin{cases} \{\}, & \text{(cleared) if } \mathsf{accum} = \mathsf{GrB\_NULL}, \\ \\ \mathbf{L}(\mathsf{s}), & \text{(unchanged) otherwise,} \end{cases}$$

or

$$\mathsf{val} = \begin{cases} \mathbf{0}(\mathsf{op}), & \text{(cleared) if } \mathsf{accum} = \mathsf{GrB\_NULL}, \\ \\ \mathsf{val} \odot \mathbf{0}(\mathsf{op}), & \text{otherwise,} \end{cases}$$

where $\mathbf{0}(\mathsf{op})$ is the identity of the monoid. The operation returns immediately with GrB_SUCCESS.

For all other cases, the internal vector and scalar used in the computation is formed ($\leftarrow$ denotes copy):

1. Vector $\widetilde{\mathbf{u}} \leftarrow \mathsf{u}$.

2. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GraphBLAS scalar case).

We are now ready to carry out the reduction and any additional associated operations. An intermediate scalar result t is computed as follows:

$$t \;=\; \bigoplus_{i \in \mathbf{ind}(\widetilde{\mathbf{u}})} \widetilde{\mathbf{u}}(i),$$

where $\oplus = \bigodot(\mathsf{op})$.

The final reduction value is computed as follows:

$$\mathbf{L}(\mathsf{s}) \leftarrow \begin{cases} \{t\}, & \text{when } \mathsf{accum} = \mathsf{GrB\_NULL} \text{ or } \tilde{s} \text{ is empty, or} \\ \\ \{\mathbf{val}(\tilde{s}) \odot t\}, & \text{otherwise;} \end{cases}$$

or

$$\mathsf{val} \leftarrow \begin{cases} t, & \text{when } \mathsf{accum} = \mathsf{GrB\_NULL}, \text{ or} \\ \\ \mathsf{val} \odot t, & \text{otherwise;} \end{cases}$$

In both GrB_BLOCKING and GrB_NONBLOCKING modes, the method exits with return value GrB_SUCCESS and the new contents of the output scalar is as defined above.

4.3.10.3 reduce: Matrix-scalar variant

Reduce all stored values into a single scalar.

C Syntax

```
// scalar value + monoid (only)
GrB_Info GrB_reduce(<type>              *val,
                    const GrB_BinaryOp   accum,
                    const GrB_Monoid     op,
                    const GrB_Matrix     A,
                    const GrB_Descriptor desc);

// GraphBLAS Scalar + monoid
GrB_Info GrB_reduce(GrB_Scalar           s,
                    const GrB_BinaryOp   accum,
                    const GrB_Monoid     op,
                    const GrB_Matrix     A,
                    const GrB_Descriptor desc);

// GraphBLAS Scalar + binary operator
GrB_Info GrB_reduce(GrB_Scalar           s,
                    const GrB_BinaryOp   accum,
                    const GrB_BinaryOp   op,
                    const GrB_Matrix     A,
                    const GrB_Descriptor desc);
```

Parameters

val or s (INOUT) Scalar to store final reduced value into. On input, the scalar provides a value that may be accumulated (optionally) with the result of the reduction operation. On output, this scalar holds the results of the operation.

accum (IN) An optional binary operator used for accumulating entries into existing (s or val) value. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The monoid ($M = \langle D, \oplus, 0 \rangle$) or binary operator ($F_b = \langle D, D, D, \oplus \rangle$) used in the reduction operation. The $\oplus$ operator must be commutative and associative; otherwise, the outcome of the operation is undefined.

A (IN) The GraphBLAS matrix on which the reduction will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description

Note: This argument is defined for consistency with the other GraphBLAS operations. There are currently no non-default field/value pairs that can be set for this operation.

Return Values

GrB_SUCCESS In blocking or non-blocking mode, the operation completed successfully, and the output scalar (s or val) is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to a respective constructor.

GrB_NULL_POINTER val pointer is NULL.

GrB_DOMAIN_MISMATCH The domains of input and output arguments are incompatible with the corresponding domains of the accumulation operator, or reduce operator.

Description

This variant of GrB_reduce computes the result of performing a reduction across all of the stored elements of an input matrix storing the result into either s or val. This corresponds to (shown here for the scalar value case only):

$$\mathsf{val} = \begin{cases} \bigoplus_{(i,j)\in\mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j), & \text{or} \\ \mathsf{val} \odot \left[\bigoplus_{(i,j)\in\mathbf{ind}(\mathsf{A})} \mathsf{A}(i,j)\right], & \text{if the the optional accumulator is specified.} \end{cases}$$

where $\bigoplus = \bigodot(\mathsf{op})$ and $\odot = \bigodot(\mathsf{accum})$.

Logically, this operation occurs in three steps:

Setup The internal matrix used in the computation is formed and its domain is tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output scalar.

One matrix argument is used in this GrB_reduce operation:

1. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{size}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{i,j})\} \rangle$

The output scalar, argument matrix, reduction operator and accumulation operator (if provided) are tested for domain compatibility as follows:

1. If accum is GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).
2. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{val})$ or $\mathbf{D}(\mathsf{s})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator, and $\mathbf{D}(\mathsf{op})$ from M (or $\mathbf{D}_{out}(\mathsf{op})$ from F_b) must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.
3. $\mathbf{D}(\mathsf{A})$ must be compatible with $\mathbf{D}(\mathsf{op})$ from M (or with $\mathbf{D}_{in_1}(\mathsf{op})$ and $\mathbf{D}_{in_2}(\mathsf{op})$ from F_b).

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_reduce ends and the domain mismatch error listed above is returned.

The number of values stored in the input, A, is checked. If there are no stored values in A, then one of the following occurs depending on the output variant:

$$\mathbf{L}(\mathsf{s}) = \begin{cases} \{\}, & \text{(cleared) if } \mathsf{accum} = \mathsf{GrB\_NULL}, \\ \\ \mathbf{L}(\mathsf{s}), & \text{(unchanged) otherwise,} \end{cases}$$

or

$$\mathsf{val} = \begin{cases} \mathbf{0}(\mathsf{op}), & \text{(cleared) if } \mathsf{accum} = \mathsf{GrB\_NULL}, \\ \\ \mathsf{val} \odot \mathbf{0}(\mathsf{op}), & \text{otherwise,} \end{cases}$$

where $\mathbf{0}(\mathsf{op})$ is the identity of the monoid. The operation returns immediately with GrB_SUCCESS.

For all other cases, the internal matrix and scalar used in the computation is formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{A}$.
2. Scalar $\tilde{s} \leftarrow \mathsf{s}$ (GraphBLAS scalar case).

We are now ready to carry out the reduce and any additional associated operations. An intermediate scalar result t is computed as follows:

$$t = \bigoplus_{(i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})} \widetilde{\mathbf{A}}(i,j),$$

where $\oplus = \bigodot(\mathsf{op})$.

The final reduction value is computed as follows:

$$\mathbf{L}(\mathsf{s}) \leftarrow \begin{cases} \{t\}, & \text{when } \mathsf{accum} = \mathsf{GrB\_NULL} \text{ or } \tilde{s} \text{ is empty, or} \\ \\ \{\mathbf{val}(\tilde{s}) \ \odot \ t\}, & \text{otherwise;} \end{cases}$$

or

$$\mathsf{val} \leftarrow \begin{cases} t, & \text{when } \mathsf{accum} = \mathsf{GrB\_NULL}\text{, or} \\ \\ \mathsf{val} \ \odot \ t, & \text{otherwise;} \end{cases}$$

In both GrB_BLOCKING and GrB_NONBLOCKING modes, the method exits with return value GrB_SUCCESS and the new contents of the output scalar is as defined above.

4.3.11 transpose: Transpose rows and columns of a matrix

This version computes a new matrix that is the transpose of the source matrix.

C Syntax

```
GrB_Info GrB_transpose(GrB_Matrix             C,
                       const GrB_Matrix       Mask,
                       const GrB_BinaryOp     accum,
                       const GrB_Matrix       A,
                       const GrB_Descriptor   desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the transpose operation. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

A (IN) The GraphBLAS matrix on which transposition will be performed.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_DIMENSION_MISMATCH mask, C and/or A dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCT is not set).

Description

GrB_transpose computes the result of performing a transpose of the input matrix: $\mathsf{C} = \mathsf{A}^T$; or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot \mathsf{A}^T$. We note that the input matrix A can itself be optionally transposed before the operation, which would cause either an assignment from A to C or an accumulation of A into C.

Logically, this operation occurs in three steps:

Setup The internal matrix and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to three matrix arguments are used in this GrB_transpose operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\}\rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\}\rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\}\rangle$

The argument matrices and accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}(\mathsf{Mask})$ must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}(\mathsf{A})$ of the input matrix.
3. If accum is not GrB_NULL, then $\mathbf{D}(\mathsf{C})$ must be compatible with $\mathbf{D}_{in_1}(\mathsf{accum})$ and $\mathbf{D}_{out}(\mathsf{accum})$ of the accumulation operator and $\mathbf{D}(\mathsf{A})$ of the input matrix must be compatible with $\mathbf{D}_{in_2}(\mathsf{accum})$ of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_transpose ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:

(a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \leq i < \mathbf{nrows}(\mathsf{C}), 0 \leq j < \mathbf{ncols}(\mathsf{C})\}\rangle$.

(b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,

i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,

ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$
$\{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.

(c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.

3. Matrix $\widetilde{\mathbf{A}} \leftarrow$ desc[GrB_INP0].GrB_TRAN ? A^T : A.

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{A}})$.
4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}})$.

If any compatibility rule above is violated, execution of **GrB_transpose** ends and the dimension mismatch error listed above is returned.

From this point forward, in **GrB_NONBLOCKING** mode, the method can optionally exit with **GrB_SUCCESS** return code and defer any computation and/or execution error codes.

We are now ready to carry out the matrix transposition and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the transpose of $\widetilde{\mathbf{A}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix

$$\widetilde{\mathbf{T}} = \langle \mathbf{D}(\mathsf{A}), \mathbf{ncols}(\widetilde{\mathbf{A}}), \mathbf{nrows}(\widetilde{\mathbf{A}}), \{(j,i,A_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{A}})\rangle$$

is created.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If $\mathsf{accum} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i,j,Z_{ij}) \forall (i,j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j) \odot \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i,j), \text{ if } (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i,j,C_{ij}) : (i,j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg\widetilde{\mathbf{M}}))\} \cup \{(i,j,Z_{ij}) : (i,j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence.

4.3.12 kronecker: Kronecker product of two matrices

Computes the Kronecker product of two matrices. The result is a matrix.

C Syntax

```
        GrB_Info GrB_kronecker(GrB_Matrix             C,
                               const GrB_Matrix       Mask,
                               const GrB_BinaryOp     accum,
                               const GrB_Semiring     op,
                               const GrB_Matrix       A,
                               const GrB_Matrix       B,
                               const GrB_Descriptor   desc);
```

```
GrB_Info GrB_kronecker(GrB_Matrix            C,
                       const GrB_Matrix      Mask,
                       const GrB_BinaryOp    accum,
                       const GrB_Monoid      op,
                       const GrB_Matrix      A,
                       const GrB_Matrix      B,
                       const GrB_Descriptor  desc);

GrB_Info GrB_kronecker(GrB_Matrix            C,
                       const GrB_Matrix      Mask,
                       const GrB_BinaryOp    accum,
                       const GrB_BinaryOp    op,
                       const GrB_Matrix      A,
                       const GrB_Matrix      B,
                       const GrB_Descriptor  desc);
```

Parameters

C (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the Kronecker product. On output, the matrix holds the results of the operation.

Mask (IN) An optional "write" mask that controls which results from this operation are stored into the output matrix C. The mask dimensions must match those of the matrix C. If the GrB_STRUCTURE descriptor is *not* set for the mask, the domain of the Mask matrix must be of type bool or any of the predefined "built-in" types in Table 3.2. If the default mask is desired (i.e., a mask that is all true with the dimensions of C), GrB_NULL should be specified.

accum (IN) An optional binary operator used for accumulating entries into existing C entries. If assignment rather than accumulation is desired, GrB_NULL should be specified.

op (IN) The semiring, monoid, or binary operator used in the element-wise "product" operation. Depending on which type is passed, the following defines the binary operator, $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \otimes \rangle$, used:

BinaryOp: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigodot(\mathsf{op}) \rangle$.

Monoid: $F_b = \langle \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \mathbf{D}(\mathsf{op}), \bigodot(\mathsf{op}) \rangle$; the identity element is ignored.

Semiring: $F_b = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{D}_{in_1}(\mathsf{op}), \mathbf{D}_{in_2}(\mathsf{op}), \bigotimes(\mathsf{op}) \rangle$; the additive monoid is ignored.

A (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the product.

B (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the product.

desc (IN) An optional operation descriptor. If a *default* descriptor is desired, GrB_NULL should be specified. Non-default field/value pairs are listed as follows:

Param	Field	Value	Description
C	GrB_OUTP	GrB_REPLACE	Output matrix C is cleared (all elements removed) before the result is stored in it.
Mask	GrB_MASK	GrB_STRUCTURE	The write mask is constructed from the structure (pattern of stored values) of the input Mask matrix. The stored values are not examined.
Mask	GrB_MASK	GrB_COMP	Use the complement of Mask.
A	GrB_INP0	GrB_TRAN	Use transpose of A for the operation.
B	GrB_INP1	GrB_TRAN	Use transpose of B for the operation.

Return Values

GrB_SUCCESS In blocking mode, the operation completed successfully. In non-blocking mode, this indicates that the compatibility tests on dimensions and domains for the input arguments passed successfully. Either way, output matrix C is ready to be used in the next method of the sequence.

GrB_PANIC Unknown internal error.

GrB_INVALID_OBJECT This is returned in any execution mode whenever one of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error. Call GrB_error() to access any error messages generated by the implementation.

GrB_OUT_OF_MEMORY Not enough memory available for the operation.

GrB_UNINITIALIZED_OBJECT One or more of the GraphBLAS objects has not been initialized by a call to new (or Matrix_dup for matrix parameters).

GrB_DIMENSION_MISMATCH Mask and/or matrix dimensions are incompatible.

GrB_DOMAIN_MISMATCH The domains of the various matrices are incompatible with the corresponding domains of the binary operator (op) or accumulation operator, or the mask's domain is not compatible with bool (in the case where desc[GrB_MASK].GrB_STRUCTURE is not set).

Description

GrB_kronecker computes the Kronecker product $\mathsf{C} = \mathsf{A} \otimes \mathsf{B}$ or, if an optional binary accumulation operator ($\odot$) is provided, $\mathsf{C} = \mathsf{C} \odot (\mathsf{A} \otimes \mathsf{B})$ (where matrices A and B can be optionally transposed).

The Kronecker product is defined as follows:

$$\mathsf{C} = \mathsf{A} \bigotimes \mathsf{B} = \begin{bmatrix} A_{0,0} \otimes \mathsf{B} & A_{0,1} \otimes \mathsf{B} & \ldots & A_{0,n_A-1} \otimes \mathsf{B} \\ A_{1,0} \otimes \mathsf{B} & A_{1,1} \otimes \mathsf{B} & \ldots & A_{1,n_A-1} \otimes \mathsf{B} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m_A-1,0} \otimes \mathsf{B} & A_{m_A-1,1} \otimes \mathsf{B} & \ldots & A_{m_A-1,n_A-1} \otimes \mathsf{B} \end{bmatrix}$$

where $\mathsf{A} : \mathbb{S}^{m_A \times n_A}$, $\mathsf{B} : \mathbb{S}^{m_B \times n_B}$, and $\mathsf{C} : \mathbb{S}^{m_A m_B \times n_A n_B}$. More explicitly, the elements of the Kronecker product are defined as

$$\mathsf{C}(i_A m_B + i_B, j_A n_B + j_B) = A_{i_A,j_A} \otimes B_{i_B,j_B},$$

where $\otimes$ is the multiplicative operator specified by the op parameter.

Logically, this operation occurs in three steps:

Setup The internal matrices and mask used in the computation are formed and their domains and dimensions are tested for compatibility.

Compute The indicated computations are carried out.

Output The result is written into the output matrix, possibly under control of a mask.

Up to four argument matrices are used in the GrB_kronecker operation:

1. $\mathsf{C} = \langle \mathbf{D}(\mathsf{C}), \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij})\} \rangle$
2. $\mathsf{Mask} = \langle \mathbf{D}(\mathsf{Mask}), \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \mathbf{L}(\mathsf{Mask}) = \{(i, j, M_{ij})\} \rangle$ (optional)
3. $\mathsf{A} = \langle \mathbf{D}(\mathsf{A}), \mathbf{nrows}(\mathsf{A}), \mathbf{ncols}(\mathsf{A}), \mathbf{L}(\mathsf{A}) = \{(i, j, A_{ij})\} \rangle$
4. $\mathsf{B} = \langle \mathbf{D}(\mathsf{B}), \mathbf{nrows}(\mathsf{B}), \mathbf{ncols}(\mathsf{B}), \mathbf{L}(\mathsf{B}) = \{(i, j, B_{ij})\} \rangle$

The argument matrices, the "product" operator (op), and the accumulation operator (if provided) are tested for domain compatibility as follows:

1. If Mask is not GrB_NULL, and desc[GrB_MASK].GrB_STRUCTURE is not set, then $\mathbf{D}$(Mask) must be from one of the pre-defined types of Table 3.2.
2. $\mathbf{D}$(A) must be compatible with $\mathbf{D}_{in_1}$(op).
3. $\mathbf{D}$(B) must be compatible with $\mathbf{D}_{in_2}$(op).
4. $\mathbf{D}$(C) must be compatible with $\mathbf{D}_{out}$(op).
5. If accum is not GrB_NULL, then $\mathbf{D}$(C) must be compatible with $\mathbf{D}_{in_1}$(accum) and $\mathbf{D}_{out}$(accum) of the accumulation operator and $\mathbf{D}_{out}$(op) of op must be compatible with $\mathbf{D}_{in_2}$(accum) of the accumulation operator.

Two domains are compatible with each other if values from one domain can be cast to values in the other domain as per the rules of the C language. In particular, domains from Table 3.2 are all compatible with each other. A domain from a user-defined type is only compatible with itself. If any compatibility rule above is violated, execution of GrB_kronecker ends and the domain mismatch error listed above is returned.

From the argument matrices, the internal matrices and mask used in the computation are formed ($\leftarrow$ denotes copy):

1. Matrix $\widetilde{\mathbf{C}} \leftarrow \mathsf{C}$.
2. Two-dimensional mask, $\widetilde{\mathbf{M}}$, is computed from argument Mask as follows:
    (a) If $\mathsf{Mask} = \mathsf{GrB\_NULL}$, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{C}), \mathbf{ncols}(\mathsf{C}), \{(i,j), \forall i,j : 0 \le i < \mathbf{nrows}(\mathsf{C}), 0 \le j < \mathbf{ncols}(\mathsf{C})\}\rangle$.
    (b) If $\mathsf{Mask} \neq \mathsf{GrB\_NULL}$,
        i. If desc[GrB_MASK].GrB_STRUCTURE is set, then $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}), \{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask})\}\rangle$,
        ii. Otherwise, $\widetilde{\mathbf{M}} = \langle \mathbf{nrows}(\mathsf{Mask}), \mathbf{ncols}(\mathsf{Mask}),$ $\{(i,j) : (i,j) \in \mathbf{ind}(\mathsf{Mask}) \wedge (\mathsf{bool})\mathsf{Mask}(i,j) = \mathsf{true}\}\rangle$.
    (c) If desc[GrB_MASK].GrB_COMP is set, then $\widetilde{\mathbf{M}} \leftarrow \neg\widetilde{\mathbf{M}}$.
3. Matrix $\widetilde{\mathbf{A}} \leftarrow \mathsf{desc[GrB\_INP0].GrB\_TRAN}\ ?\ \mathsf{A}^T : \mathsf{A}$.
4. Matrix $\widetilde{\mathbf{B}} \leftarrow \mathsf{desc[GrB\_INP1].GrB\_TRAN}\ ?\ \mathsf{B}^T : \mathsf{B}$.

The internal matrices and masks are checked for dimension compatibility. The following conditions must hold:

1. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{M}})$.
2. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{M}})$.
3. $\mathbf{nrows}(\widetilde{\mathbf{C}}) = \mathbf{nrows}(\widetilde{\mathbf{A}}) \cdot \mathbf{nrows}(\widetilde{\mathbf{B}})$.
4. $\mathbf{ncols}(\widetilde{\mathbf{C}}) = \mathbf{ncols}(\widetilde{\mathbf{A}}) \cdot \mathbf{ncols}(\widetilde{\mathbf{B}})$.

If any compatibility rule above is violated, execution of GrB_kronecker ends and the dimension mismatch error listed above is returned.

From this point forward, in GrB_NONBLOCKING mode, the method can optionally exit with GrB_SUCCESS return code and defer any computation and/or execution error codes.

We are now ready to carry out the Kronecker product and any additional associated operations. We describe this in terms of two intermediate matrices:

- $\widetilde{\mathbf{T}}$: The matrix holding the Kronecker product of matrices $\widetilde{\mathbf{A}}$ and $\widetilde{\mathbf{B}}$.
- $\widetilde{\mathbf{Z}}$: The matrix holding the result after application of the (optional) accumulation operator.

The intermediate matrix $\widetilde{\mathbf{T}} = \langle \mathbf{D}_{out}(\mathsf{op}), \mathbf{nrows}(\widetilde{\mathbf{A}}) \times \mathbf{nrows}(\widetilde{\mathbf{B}}), \mathbf{ncols}(\widetilde{\mathbf{A}}) \times \mathbf{ncols}(\widetilde{\mathbf{B}}), \{(i, j, T_{ij})$ where $i = i_A \cdot m_B + i_B,\ j = j_A \cdot n_B + j_B,\ \forall\ (i_A, j_A) = \mathbf{ind}(\widetilde{\mathbf{A}}),\ (i_B, j_B) = \mathbf{ind}(\widetilde{\mathbf{B}})\rangle$ is created. The value of each of its elements is computed by

$$T_{i_A \cdot m_B + i_B,\ j_A \cdot n_B + j_B} = \widetilde{\mathbf{A}}(i_A, j_A) \otimes \widetilde{\mathbf{B}}(i_B, j_B)),$$

where $\otimes$ is the multiplicative operator specified by the op parameter.

The intermediate matrix $\widetilde{\mathbf{Z}}$ is created as follows, using what is called a *standard matrix accumulate*:

- If accum = GrB_NULL, then $\widetilde{\mathbf{Z}} = \widetilde{\mathbf{T}}$.
- If accum is a binary operator, then $\widetilde{\mathbf{Z}}$ is defined as

$$\widetilde{\mathbf{Z}} = \langle \mathbf{D}_{out}(\mathsf{accum}), \mathbf{nrows}(\widetilde{\mathbf{C}}), \mathbf{ncols}(\widetilde{\mathbf{C}}), \{(i, j, Z_{ij}) \forall (i, j) \in \mathbf{ind}(\widetilde{\mathbf{C}}) \cup \mathbf{ind}(\widetilde{\mathbf{T}})\}\rangle.$$

  The values of the elements of $\widetilde{\mathbf{Z}}$ are computed based on the relationships between the sets of indices in $\widetilde{\mathbf{C}}$ and $\widetilde{\mathbf{T}}$.

$$Z_{ij} = \widetilde{\mathbf{C}}(i, j) \odot \widetilde{\mathbf{T}}(i, j),\ \text{if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}})),$$

$$Z_{ij} = \widetilde{\mathbf{C}}(i, j),\ \text{if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{C}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

$$Z_{ij} = \widetilde{\mathbf{T}}(i, j),\ \text{if } (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{T}}) - (\mathbf{ind}(\widetilde{\mathbf{T}}) \cap \mathbf{ind}(\widetilde{\mathbf{C}}))),$$

  where $\odot = \bigodot(\mathsf{accum})$, and the difference operator refers to set difference.

Finally, the set of output values that make up matrix $\widetilde{\mathbf{Z}}$ are written into the final result matrix C, using what is called a *standard matrix mask and replace*. This is carried out under control of the mask which acts as a "write mask".

- If desc[GrB_OUTP].GrB_REPLACE is set, then any values in C on input to this operation are deleted and the content of the new output matrix, C, is defined as,

$$\mathbf{L}(\mathsf{C}) = \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

- If desc[GrB_OUTP].GrB_REPLACE is not set, the elements of $\widetilde{\mathbf{Z}}$ indicated by the mask are copied into the result matrix, C, and elements of C that fall outside the set indicated by the mask are unchanged:

$$\mathbf{L}(\mathsf{C}) = \{(i, j, C_{ij}) : (i, j) \in (\mathbf{ind}(\mathsf{C}) \cap \mathbf{ind}(\neg \widetilde{\mathbf{M}}))\} \cup \{(i, j, Z_{ij}) : (i, j) \in (\mathbf{ind}(\widetilde{\mathbf{Z}}) \cap \mathbf{ind}(\widetilde{\mathbf{M}}))\}.$$

In GrB_BLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above and fully computed. In GrB_NONBLOCKING mode, the method exits with return value GrB_SUCCESS and the new content of matrix C is as defined above but may not be fully computed. However, it can be used in the next GraphBLAS method call in a sequence. s

Chapter 5

Nonpolymorphic interface

Each polymorphic GraphBLAS method (those with multiple parameter signatures under the same name) has a corresponding set of long-name forms that are specific to each parameter signature. That is show in Tables 5.1 through 5.11.

Table 5.1: Long-name, nonpolymorphic form of GraphBLAS methods.

Polymorphic signature	Nonpolymorphic signature
GrB_Monoid_new(GrB_Monoid*,...,bool)	GrB_Monoid_new_BOOL(GrB_Monoid*,GrB_BinaryOp,bool)
GrB_Monoid_new(GrB_Monoid*,...,int8_t)	GrB_Monoid_new_INT8(GrB_Monoid*,GrB_BinaryOp,int8_t)
GrB_Monoid_new(GrB_Monoid*,...,uint8_t)	GrB_Monoid_new_UINT8(GrB_Monoid*,GrB_BinaryOp,uint8_t)
GrB_Monoid_new(GrB_Monoid*,...,int16_t)	GrB_Monoid_new_INT16(GrB_Monoid*,GrB_BinaryOp,int16_t)
GrB_Monoid_new(GrB_Monoid*,...,uint16_t)	GrB_Monoid_new_UINT16(GrB_Monoid*,GrB_BinaryOp,uint16_t)
GrB_Monoid_new(GrB_Monoid*,...,int32_t)	GrB_Monoid_new_INT32(GrB_Monoid*,GrB_BinaryOp,int32_t)
GrB_Monoid_new(GrB_Monoid*,...,uint32_t)	GrB_Monoid_new_UINT32(GrB_Monoid*,GrB_BinaryOp,uint32_t)
GrB_Monoid_new(GrB_Monoid*,...,int64_t)	GrB_Monoid_new_INT64(GrB_Monoid*,GrB_BinaryOp,int64_t)
GrB_Monoid_new(GrB_Monoid*,...,uint64_t)	GrB_Monoid_new_UINT64(GrB_Monoid*,GrB_BinaryOp,uint64_t)
GrB_Monoid_new(GrB_Monoid*,...,float)	GrB_Monoid_new_FP32(GrB_Monoid*,GrB_BinaryOp,float)
GrB_Monoid_new(GrB_Monoid*,...,double)	GrB_Monoid_new_FP64(GrB_Monoid*,GrB_BinaryOp,double)
GrB_Monoid_new(GrB_Monoid*,...,*other*)	GrB_Monoid_new_UDT(GrB_Monoid*,GrB_BinaryOp,void*)

Table 5.2: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_Scalar_setElement(. . . , bool,. . .)	GrB_Scalar_setElement_BOOL(. . . , bool,. . .)
GrB_Scalar_setElement(. . . , int8_t,. . .)	GrB_Scalar_setElement_INT8(. . . , int8_t,. . .)
GrB_Scalar_setElement(. . . , uint8_t,. . .)	GrB_Scalar_setElement_UINT8(. . . , uint8_t,. . .)
GrB_Scalar_setElement(. . . , int16_t,. . .)	GrB_Scalar_setElement_INT16(. . . , int16_t,. . .)
GrB_Scalar_setElement(. . . , uint16_t,. . .)	GrB_Scalar_setElement_UINT16(. . . , uint16_t,. . .)
GrB_Scalar_setElement(. . . , int32_t,. . .)	GrB_Scalar_setElement_INT32(. . . , int32_t,. . .)
GrB_Scalar_setElement(. . . , uint32_t,. . .)	GrB_Scalar_setElement_UINT32(. . . , uint32_t,. . .)
GrB_Scalar_setElement(. . . , int64_t,. . .)	GrB_Scalar_setElement_INT64(. . . , int64_t,. . .)
GrB_Scalar_setElement(. . . , uint64_t,. . .)	GrB_Scalar_setElement_UINT64(. . . , uint64_t,. . .)
GrB_Scalar_setElement(. . . , float,. . .)	GrB_Scalar_setElement_FP32(. . . , float,. . .)
GrB_Scalar_setElement(. . . , double,. . .)	GrB_Scalar_setElement_FP64(. . . , double,. . .)
GrB_Scalar_setElement(. . . ,*other*,. . .)	GrB_Scalar_setElement_UDT(. . . ,const void*,. . .)
GrB_Scalar_extractElement(bool*,. . .)	GrB_Scalar_extractElement_BOOL(bool*,. . .)
GrB_Scalar_extractElement(int8_t*,. . .)	GrB_Scalar_extractElement_INT8(int8_t*,. . .)
GrB_Scalar_extractElement(uint8_t*,. . .)	GrB_Scalar_extractElement_UINT8(uint8_t*,. . .)
GrB_Scalar_extractElement(int16_t*,. . .)	GrB_Scalar_extractElement_INT16(int16_t*,. . .)
GrB_Scalar_extractElement(uint16_t*,. . .)	GrB_Scalar_extractElement_UINT16(uint16_t*,. . .)
GrB_Scalar_extractElement(int32_t*,. . .)	GrB_Scalar_extractElement_INT32(int32_t*,. . .)
GrB_Scalar_extractElement(uint32_t*,. . .)	GrB_Scalar_extractElement_UINT32(uint32_t*,. . .)
GrB_Scalar_extractElement(int64_t*,. . .)	GrB_Scalar_extractElement_INT64(int64_t*,. . .)
GrB_Scalar_extractElement(uint64_t*,. . .)	GrB_Scalar_extractElement_UINT64(uint64_t*,. . .)
GrB_Scalar_extractElement(float*,. . .)	GrB_Scalar_extractElement_FP32(float*,. . .)
GrB_Scalar_extractElement(double*,. . .)	GrB_Scalar_extractElement_FP64(double*,. . .)
GrB_Scalar_extractElement(*other**,. . .)	GrB_Scalar_extractElement_UDT(void*,. . .)

Table 5.3: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_Vector_build(...,const bool*,...)	GrB_Vector_build_BOOL(...,const bool*,...)
GrB_Vector_build(...,const int8_t*,...)	GrB_Vector_build_INT8(...,const int8_t*,...)
GrB_Vector_build(...,const uint8_t*,...)	GrB_Vector_build_UINT8(...,const uint8_t*,...)
GrB_Vector_build(...,const int16_t*,...)	GrB_Vector_build_INT16(...,const int16_t*,...)
GrB_Vector_build(...,const uint16_t*,...)	GrB_Vector_build_UINT16(...,const uint16_t*,...)
GrB_Vector_build(...,const int32_t*,...)	GrB_Vector_build_INT32(...,const int32_t*,...)
GrB_Vector_build(...,const uint32_t*,...)	GrB_Vector_build_UINT32(...,const uint32_t*,...)
GrB_Vector_build(...,const int64_t*,...)	GrB_Vector_build_INT64(...,const int64_t*,...)
GrB_Vector_build(...,const uint64_t*,...)	GrB_Vector_build_UINT64(...,const uint64_t*,...)
GrB_Vector_build(...,const float*,...)	GrB_Vector_build_FP32(...,const float*,...)
GrB_Vector_build(...,const double*,...)	GrB_Vector_build_FP64(...,const double*,...)
GrB_Vector_build(...,const *other**,...)	GrB_Vector_build_UDT(...,const void*,...)
GrB_Vector_setElement(...,GrB_Scalar,...)	GrB_Vector_setElement_Scalar(...,const GrB_Scalar,...)
GrB_Vector_setElement(...,bool,...)	GrB_Vector_setElement_BOOL(..., bool,...)
GrB_Vector_setElement(...,int8_t,...)	GrB_Vector_setElement_INT8(..., int8_t,...)
GrB_Vector_setElement(...,uint8_t,...)	GrB_Vector_setElement_UINT8(..., uint8_t,...)
GrB_Vector_setElement(...,int16_t,...)	GrB_Vector_setElement_INT16(..., int16_t,...)
GrB_Vector_setElement(...,uint16_t,...)	GrB_Vector_setElement_UINT16(..., uint16_t,...)
GrB_Vector_setElement(...,int32_t,...)	GrB_Vector_setElement_INT32(..., int32_t,...)
GrB_Vector_setElement(...,uint32_t,...)	GrB_Vector_setElement_UINT32(..., uint32_t,...)
GrB_Vector_setElement(...,int64_t,...)	GrB_Vector_setElement_INT64(..., int64_t,...)
GrB_Vector_setElement(...,uint64_t,...)	GrB_Vector_setElement_UINT64(..., uint64_t,...)
GrB_Vector_setElement(...,float,...)	GrB_Vector_setElement_FP32(..., float,...)
GrB_Vector_setElement(...,double,...)	GrB_Vector_setElement_FP64(..., double,...)
GrB_Vector_setElement(...,*other*,...)	GrB_Vector_setElement_UDT(...,const void*,...)
GrB_Vector_extractElement(GrB_Scalar,...)	GrB_Vector_extractElement_Scalar(GrB_Scalar,...)
GrB_Vector_extractElement(bool*,...)	GrB_Vector_extractElement_BOOL(bool*,...)
GrB_Vector_extractElement(int8_t*,...)	GrB_Vector_extractElement_INT8(int8_t*,...)
GrB_Vector_extractElement(uint8_t*,...)	GrB_Vector_extractElement_UINT8(uint8_t*,...)
GrB_Vector_extractElement(int16_t*,...)	GrB_Vector_extractElement_INT16(int16_t*,...)
GrB_Vector_extractElement(uint16_t*,...)	GrB_Vector_extractElement_UINT16(uint16_t*,...)
GrB_Vector_extractElement(int32_t*,...)	GrB_Vector_extractElement_INT32(int32_t*,...)
GrB_Vector_extractElement(uint32_t*,...)	GrB_Vector_extractElement_UINT32(uint32_t*,...)
GrB_Vector_extractElement(int64_t*,...)	GrB_Vector_extractElement_INT64(int64_t*,...)
GrB_Vector_extractElement(uint64_t*,...)	GrB_Vector_extractElement_UINT64(uint64_t*,...)
GrB_Vector_extractElement(float*,...)	GrB_Vector_extractElement_FP32(float*,...)
GrB_Vector_extractElement(double*,...)	GrB_Vector_extractElement_FP64(double*,...)
GrB_Vector_extractElement(*other**,...)	GrB_Vector_extractElement_UDT(void*,...)
GrB_Vector_extractTuples(...,bool*,...)	GrB_Vector_extractTuples_BOOL(..., bool*,...)
GrB_Vector_extractTuples(...,int8_t*,...)	GrB_Vector_extractTuples_INT8(..., int8_t*,...)
GrB_Vector_extractTuples(...,uint8_t*,...)	GrB_Vector_extractTuples_UINT8(..., uint8_t*,...)
GrB_Vector_extractTuples(...,int16_t*,...)	GrB_Vector_extractTuples_INT16(..., int16_t*,...)
GrB_Vector_extractTuples(...,uint16_t*,...)	GrB_Vector_extractTuples_UINT16(..., uint16_t*,...)
GrB_Vector_extractTuples(...,int32_t*,...)	GrB_Vector_extractTuples_INT32(..., int32_t*,...)
GrB_Vector_extractTuples(...,uint32_t*,...)	GrB_Vector_extractTuples_UINT32(..., uint32_t*,...)
GrB_Vector_extractTuples(...,int64_t*,...)	GrB_Vector_extractTuples_INT64(..., int64_t*,...)
GrB_Vector_extractTuples(...,uint64_t*,...)	GrB_Vector_extractTuples_UINT64(..., uint64_t*,...)
GrB_Vector_extractTuples(...,float*,...)	GrB_Vector_extractTuples_FP32(..., float*,...)
GrB_Vector_extractTuples(...,double*,...)	GrB_Vector_extractTuples_FP64(..., double*,...)
GrB_Vector_extractTuples(...,*other**,...)	GrB_Vector_extractTuples_UDT(..., void*,...)

Table 5.4: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_Matrix_build(...,const bool*,...)	GrB_Matrix_build_BOOL(...,const bool*,...)
GrB_Matrix_build(...,const int8_t*,...)	GrB_Matrix_build_INT8(...,const int8_t*,...)
GrB_Matrix_build(...,const uint8_t*,...)	GrB_Matrix_build_UINT8(...,const uint8_t*,...)
GrB_Matrix_build(...,const int16_t*,...)	GrB_Matrix_build_INT16(...,const int16_t*,...)
GrB_Matrix_build(...,const uint16_t*,...)	GrB_Matrix_build_UINT16(...,const uint16_t*,...)
GrB_Matrix_build(...,const int32_t*,...)	GrB_Matrix_build_INT32(...,const int32_t*,...)
GrB_Matrix_build(...,const uint32_t*,...)	GrB_Matrix_build_UINT32(...,const uint32_t*,...)
GrB_Matrix_build(...,const int64_t*,...)	GrB_Matrix_build_INT64(...,const int64_t*,...)
GrB_Matrix_build(...,const uint64_t*,...)	GrB_Matrix_build_UINT64(...,const uint64_t*,...)
GrB_Matrix_build(...,const float*,...)	GrB_Matrix_build_FP32(...,const float*,...)
GrB_Matrix_build(...,const double*,...)	GrB_Matrix_build_FP64(...,const double*,...)
GrB_Matrix_build(...,const *other**,...)	GrB_Matrix_build_UDT(...,const void*,...)
GrB_Matrix_setElement(...,GrB_Scalar,...)	GrB_Matrix_setElement_Scalar(...,const GrB_Scalar,...)
GrB_Matrix_setElement(...,bool,...)	GrB_Matrix_setElement_BOOL(..., bool,...)
GrB_Matrix_setElement(...,int8_t,...)	GrB_Matrix_setElement_INT8(..., int8_t,...)
GrB_Matrix_setElement(...,uint8_t,...)	GrB_Matrix_setElement_UINT8(..., uint8_t,...)
GrB_Matrix_setElement(...,int16_t,...)	GrB_Matrix_setElement_INT16(..., int16_t,...)
GrB_Matrix_setElement(...,uint16_t,...)	GrB_Matrix_setElement_UINT16(..., uint16_t,...)
GrB_Matrix_setElement(...,int32_t,...)	GrB_Matrix_setElement_INT32(..., int32_t,...)
GrB_Matrix_setElement(...,uint32_t,...)	GrB_Matrix_setElement_UINT32(..., uint32_t,...)
GrB_Matrix_setElement(...,int64_t,...)	GrB_Matrix_setElement_INT64(..., int64_t,...)
GrB_Matrix_setElement(...,uint64_t,...)	GrB_Matrix_setElement_UINT64(..., uint64_t,...)
GrB_Matrix_setElement(...,float,...)	GrB_Matrix_setElement_FP32(..., float,...)
GrB_Matrix_setElement(...,double,...)	GrB_Matrix_setElement_FP64(..., double,...)
GrB_Matrix_setElement(...,*other*,...)	GrB_Matrix_setElement_UDT(...,const void*,...)
GrB_Matrix_extractElement(GrB_Scalar,...)	GrB_Matrix_extractElement_Scalar(GrB_Scalar,...)
GrB_Matrix_extractElement(bool*,...)	GrB_Matrix_extractElement_BOOL(bool*,...)
GrB_Matrix_extractElement(int8_t*,...)	GrB_Matrix_extractElement_INT8(int8_t*,...)
GrB_Matrix_extractElement(uint8_t*,...)	GrB_Matrix_extractElement_UINT8(uint8_t*,...)
GrB_Matrix_extractElement(int16_t*,...)	GrB_Matrix_extractElement_INT16(int16_t*,...)
GrB_Matrix_extractElement(uint16_t*,...)	GrB_Matrix_extractElement_UINT16(uint16_t*,...)
GrB_Matrix_extractElement(int32_t*,...)	GrB_Matrix_extractElement_INT32(int32_t*,...)
GrB_Matrix_extractElement(uint32_t*,...)	GrB_Matrix_extractElement_UINT32(uint32_t*,...)
GrB_Matrix_extractElement(int64_t*,...)	GrB_Matrix_extractElement_INT64(int64_t*,...)
GrB_Matrix_extractElement(uint64_t*,...)	GrB_Matrix_extractElement_UINT64(uint64_t*,...)
GrB_Matrix_extractElement(float*,...)	GrB_Matrix_extractElement_FP32(float*,...)
GrB_Matrix_extractElement(double*,...)	GrB_Matrix_extractElement_FP64(double*,...)
GrB_Matrix_extractElement(*other*,...)	GrB_Matrix_extractElement_UDT(void*,...)
GrB_Matrix_extractTuples(..., bool*,...)	GrB_Matrix_extractTuples_BOOL(..., bool*,...)
GrB_Matrix_extractTuples(..., int8_t*,...)	GrB_Matrix_extractTuples_INT8(..., int8_t*,...)
GrB_Matrix_extractTuples(..., uint8_t*,...)	GrB_Matrix_extractTuples_UINT8(..., uint8_t*,...)
GrB_Matrix_extractTuples(..., int16_t*,...)	GrB_Matrix_extractTuples_INT16(..., int16_t*,...)
GrB_Matrix_extractTuples(..., uint16_t*,...)	GrB_Matrix_extractTuples_UINT16(..., uint16_t*,...)
GrB_Matrix_extractTuples(..., int32_t*,...)	GrB_Matrix_extractTuples_INT32(..., int32_t*,...)
GrB_Matrix_extractTuples(..., uint32_t*,...)	GrB_Matrix_extractTuples_UINT32(..., uint32_t*,...)
GrB_Matrix_extractTuples(..., int64_t*,...)	GrB_Matrix_extractTuples_INT64(..., int64_t*,...)
GrB_Matrix_extractTuples(..., uint64_t*,...)	GrB_Matrix_extractTuples_UINT64(..., uint64_t*,...)
GrB_Matrix_extractTuples(..., float*,...)	GrB_Matrix_extractTuples_FP32(..., float*,...)
GrB_Matrix_extractTuples(..., double*,...)	GrB_Matrix_extractTuples_FP64(..., double*,...)
GrB_Matrix_extractTuples(...,*other**,...)	GrB_Matrix_extractTuples_UDT(..., void*,...)

Table 5.5: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_Matrix_import(...,const bool*,...)	GrB_Matrix_import_BOOL(...,const bool*,...)
GrB_Matrix_import(...,const int8_t*,...)	GrB_Matrix_import_INT8(...,const int8_t*,...)
GrB_Matrix_import(...,const uint8_t*,...)	GrB_Matrix_import_UINT8(...,const uint8_t*,...)
GrB_Matrix_import(...,const int16_t*,...)	GrB_Matrix_import_INT16(...,const int16_t*,...)
GrB_Matrix_import(...,const uint16_t*,...)	GrB_Matrix_import_UINT16(...,const uint16_t*,...)
GrB_Matrix_import(...,const int32_t*,...)	GrB_Matrix_import_INT32(...,const int32_t*,...)
GrB_Matrix_import(...,const uint32_t*,...)	GrB_Matrix_import_UINT32(...,const uint32_t*,...)
GrB_Matrix_import(...,const int64_t*,...)	GrB_Matrix_import_INT64(...,const int64_t*,...)
GrB_Matrix_import(...,const uint64_t*,...)	GrB_Matrix_import_UINT64(...,const uint64_t*,...)
GrB_Matrix_import(...,const float*,...)	GrB_Matrix_import_FP32(...,const float*,...)
GrB_Matrix_import(...,const double*,...)	GrB_Matrix_import_FP64(...,const double*,...)
GrB_Matrix_import(...,const *other*,...)	GrB_Matrix_import_UDT(...,const void*,...)
GrB_Matrix_export(...,bool*,...)	GrB_Matrix_export_BOOL(...,bool*,...)
GrB_Matrix_export(...,int8_t*,...)	GrB_Matrix_export_INT8(...,int8_t*,...)
GrB_Matrix_export(...,uint8_t*,...)	GrB_Matrix_export_UINT8(...,uint8_t*,...)
GrB_Matrix_export(...,int16_t*,...)	GrB_Matrix_export_INT16(...,int16_t*,...)
GrB_Matrix_export(...,uint16_t*,...)	GrB_Matrix_export_UINT16(...,uint16_t*,...)
GrB_Matrix_export(...,int32_t*,...)	GrB_Matrix_export_INT32(...,int32_t*,...)
GrB_Matrix_export(...,uint32_t*,...)	GrB_Matrix_export_UINT32(...,uint32_t*,...)
GrB_Matrix_export(...,int64_t*,...)	GrB_Matrix_export_INT64(...,int64_t*,...)
GrB_Matrix_export(...,uint64_t*,...)	GrB_Matrix_export_UINT64(...,uint64_t*,...)
GrB_Matrix_export(...,float*,...)	GrB_Matrix_export_FP32(...,float*,...)
GrB_Matrix_export(...,double*,...)	GrB_Matrix_export_FP64(...,double*,...)
GrB_Matrix_export(...,*other*,...)	GrB_Matrix_export_UDT(...,void*,...)
GrB_free(GrB_Type*)	GrB_Type_free(GrB_Type*)
GrB_free(GrB_UnaryOp*)	GrB_UnaryOp_free(GrB_UnaryOp*)
GrB_free(GrB_IndexUnaryOp*)	GrB_IndexUnaryOp_free(GrB_IndexUnaryOp*)
GrB_free(GrB_BinaryOp*)	GrB_BinaryOp_free(GrB_BinaryOp*)
GrB_free(GrB_Monoid*)	GrB_Monoid_free(GrB_Monoid*)
GrB_free(GrB_Semiring*)	GrB_Semiring_free(GrB_Semiring*)
GrB_free(GrB_Scalar*)	GrB_Scalar_free(GrB_Scalar*)
GrB_free(GrB_Vector*)	GrB_Vector_free(GrB_Vector*)
GrB_free(GrB_Matrix*)	GrB_Matrix_free(GrB_Matrix*)
GrB_free(GrB_Descriptor*)	GrB_Descriptor_free(GrB_Descriptor*)
GrB_wait(GrB_Type, GrB_WaitMode)	GrB_Type_wait(GrB_Type, GrB_WaitMode)
GrB_wait(GrB_UnaryOp, GrB_WaitMode)	GrB_UnaryOp_wait(GrB_UnaryOp, GrB_WaitMode)
GrB_wait(GrB_IndexUnaryOp, GrB_WaitMode)	GrB_IndexUnaryOp_wait(GrB_IndexUnaryOp, GrB_WaitMode)
GrB_wait(GrB_BinaryOp, GrB_WaitMode)	GrB_BinaryOp_wait(GrB_BinaryOp, GrB_WaitMode)
GrB_wait(GrB_Monoid, GrB_WaitMode)	GrB_Monoid_wait(GrB_Monoid, GrB_WaitMode)
GrB_wait(GrB_Semiring, GrB_WaitMode)	GrB_Semiring_wait(GrB_Semiring, GrB_WaitMode)
GrB_wait(GrB_Scalar, GrB_WaitMode)	GrB_Scalar_wait(GrB_Scalar, GrB_WaitMode)
GrB_wait(GrB_Vector, GrB_WaitMode)	GrB_Vector_wait(GrB_Vector, GrB_WaitMode)
GrB_wait(GrB_Matrix, GrB_WaitMode)	GrB_Matrix_wait(GrB_Matrix, GrB_WaitMode)
GrB_wait(GrB_Descriptor, GrB_WaitMode)	GrB_Descriptor_wait(GrB_Descriptor, GrB_WaitMode)
GrB_error(const char**, const GrB_Type)	GrB_Type_error(const char**, const GrB_Type)
GrB_error(const char**, const GrB_UnaryOp)	GrB_UnaryOp_error(const char**, const GrB_UnaryOp)
GrB_error(const char**, const GrB_IndexUnaryOp)	GrB_IndexUnaryOp_error(const char**, const GrB_IndexUnaryOp)
GrB_error(const char**, const GrB_BinaryOp)	GrB_BinaryOp_error(const char**, const GrB_BinaryOp)
GrB_error(const char**, const GrB_Monoid)	GrB_Monoid_error(const char**, const GrB_Monoid)
GrB_error(const char**, const GrB_Semiring)	GrB_Semiring_error(const char**, const GrB_Semiring)
GrB_error(const char**, const GrB_Scalar)	GrB_Scalar_error(const char**, const GrB_Scalar)
GrB_error(const char**, const GrB_Vector)	GrB_Vector_error(const char**, const GrB_Vector)
GrB_error(const char**, const GrB_Matrix)	GrB_Matrix_error(const char**, const GrB_Matrix)
GrB_error(const char**, const GrB_Descriptor)	GrB_Descriptor_error(const char**, const GrB_Descriptor)

Table 5.6: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_eWiseMult(GrB_Vector,...,GrB_Semiring,...)	GrB_Vector_eWiseMult_Semiring(GrB_Vector,...,GrB_Semiring,...)
GrB_eWiseMult(GrB_Vector,...,GrB_Monoid,...)	GrB_Vector_eWiseMult_Monoid(GrB_Vector,...,GrB_Monoid,...)
GrB_eWiseMult(GrB_Vector,...,GrB_BinaryOp,...)	GrB_Vector_eWiseMult_BinaryOp(GrB_Vector,...,GrB_BinaryOp,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Semiring,...)	GrB_Matrix_eWiseMult_Semiring(GrB_Matrix,...,GrB_Semiring,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_Monoid,...)	GrB_Matrix_eWiseMult_Monoid(GrB_Matrix,...,GrB_Monoid,...)
GrB_eWiseMult(GrB_Matrix,...,GrB_BinaryOp,...)	GrB_Matrix_eWiseMult_BinaryOp(GrB_Matrix,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Vector,...,GrB_Semiring,...)	GrB_Vector_eWiseAdd_Semiring(GrB_Vector,...,GrB_Semiring,...)
GrB_eWiseAdd(GrB_Vector,...,GrB_Monoid,...)	GrB_Vector_eWiseAdd_Monoid(GrB_Vector,...,GrB_Monoid,...)
GrB_eWiseAdd(GrB_Vector,...,GrB_BinaryOp,...)	GrB_Vector_eWiseAdd_BinaryOp(GrB_Vector,...,GrB_BinaryOp,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Semiring,...)	GrB_Matrix_eWiseAdd_Semiring(GrB_Matrix,...,GrB_Semiring,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_Monoid,...)	GrB_Matrix_eWiseAdd_Monoid(GrB_Matrix,...,GrB_Monoid,...)
GrB_eWiseAdd(GrB_Matrix,...,GrB_BinaryOp,...)	GrB_Matrix_eWiseAdd_BinaryOp(GrB_Matrix,...,GrB_BinaryOp,...)
GrB_extract(GrB_Vector,...,GrB_Vector,...)	GrB_Vector_extract(GrB_Vector,...,GrB_Vector,...)
GrB_extract(GrB_Matrix,...,GrB_Matrix,...)	GrB_Matrix_extract(GrB_Matrix,...,GrB_Matrix,...)
GrB_extract(GrB_Vector,...,GrB_Matrix,...)	GrB_Col_extract(GrB_Vector,...,GrB_Matrix,...)
GrB_assign(GrB_Vector,...,GrB_Vector,...)	GrB_Vector_assign(GrB_Vector,...,GrB_Vector,...)
GrB_assign(GrB_Matrix,...,GrB_Matrix,...)	GrB_Matrix_assign(GrB_Matrix,...,GrB_Matrix,...)
GrB_assign(GrB_Matrix,...,GrB_Vector,const GrB_Index*,...)	GrB_Col_assign(GrB_Matrix,...,GrB_Vector,const GrB_Index*,...)
GrB_assign(GrB_Matrix,...,GrB_Vector,GrB_Index,...)	GrB_Row_assign(GrB_Matrix,...,GrB_Vector,GrB_Index,...)
GrB_assign(GrB_Vector,...,GrB_Scalar,...)	GrB_Vector_assign_Scalar(GrB_Vector,...,const GrB_Scalar,...)
GrB_assign(GrB_Vector,...,bool,...)	GrB_Vector_assign_BOOL(GrB_Vector,..., bool,...)
GrB_assign(GrB_Vector,...,int8_t,...)	GrB_Vector_assign_INT8(GrB_Vector,..., int8_t,...)
GrB_assign(GrB_Vector,...,uint8_t,...)	GrB_Vector_assign_UINT8(GrB_Vector,..., uint8_t,...)
GrB_assign(GrB_Vector,...,int16_t,...)	GrB_Vector_assign_INT16(GrB_Vector,..., int16_t,...)
GrB_assign(GrB_Vector,...,uint16_t,...)	GrB_Vector_assign_UINT16(GrB_Vector,..., uint16_t,...)
GrB_assign(GrB_Vector,...,int32_t,...)	GrB_Vector_assign_INT32(GrB_Vector,..., int32_t,...)
GrB_assign(GrB_Vector,...,uint32_t,...)	GrB_Vector_assign_UINT32(GrB_Vector,..., uint32_t,...)
GrB_assign(GrB_Vector,...,int64_t,...)	GrB_Vector_assign_INT64(GrB_Vector,..., int64_t,...)
GrB_assign(GrB_Vector,...,uint64_t,...)	GrB_Vector_assign_UINT64(GrB_Vector,..., uint64_t,...)
GrB_assign(GrB_Vector,...,float,...)	GrB_Vector_assign_FP32(GrB_Vector,..., float,...)
GrB_assign(GrB_Vector,...,double,...)	GrB_Vector_assign_FP64(GrB_Vector,..., double,...)
GrB_assign(GrB_Vector,...,*other*,...)	GrB_Vector_assign_UDT(GrB_Vector,...,const void*,...)
GrB_assign(GrB_Matrix,...,GrB_Scalar,...)	GrB_Matrix_assign_Scalar(GrB_Matrix,...,const GrB_Scalar,...)
GrB_assign(GrB_Matrix,...,bool,...)	GrB_Matrix_assign_BOOL(GrB_Matrix,..., bool,...)
GrB_assign(GrB_Matrix,...,int8_t,...)	GrB_Matrix_assign_INT8(GrB_Matrix,..., int8_t,...)
GrB_assign(GrB_Matrix,...,uint8_t,...)	GrB_Matrix_assign_UINT8(GrB_Matrix,..., uint8_t,...)
GrB_assign(GrB_Matrix,...,int16_t,...)	GrB_Matrix_assign_INT16(GrB_Matrix,..., int16_t,...)
GrB_assign(GrB_Matrix,...,uint16_t,...)	GrB_Matrix_assign_UINT16(GrB_Matrix,..., uint16_t,...)
GrB_assign(GrB_Matrix,...,int32_t,...)	GrB_Matrix_assign_INT32(GrB_Matrix,..., int32_t,...)
GrB_assign(GrB_Matrix,...,uint32_t,...)	GrB_Matrix_assign_UINT32(GrB_Matrix,..., uint32_t,...)
GrB_assign(GrB_Matrix,...,int64_t,...)	GrB_Matrix_assign_INT64(GrB_Matrix,..., int64_t,...)
GrB_assign(GrB_Matrix,...,uint64_t,...)	GrB_Matrix_assign_UINT64(GrB_Matrix,..., uint64_t,...)
GrB_assign(GrB_Matrix,...,float,...)	GrB_Matrix_assign_FP32(GrB_Matrix,..., float,...)
GrB_assign(GrB_Matrix,...,double,...)	GrB_Matrix_assign_FP64(GrB_Matrix,..., double,...)
GrB_assign(GrB_Matrix,...,*other*,...)	GrB_Matrix_assign_UDT(GrB_Matrix,...,const void*,...)

Table 5.7: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_apply(GrB_Vector,...,GrB_UnaryOp,GrB_Vector,...)	GrB_Vector_apply(GrB_Vector,...,GrB_UnaryOp,GrB_Vector,...)
GrB_apply(GrB_Matrix,...,GrB_UnaryOp,GrB_Matrix,...)	GrB_Matrix_apply(GrB_Matrix,...,GrB_UnaryOp,GrB_Matrix,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Scalar,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_Scalar(GrB_Vector,...,GrB_BinaryOp,GrB_Scalar,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,bool,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_BOOL(GrB_Vector,...,GrB_BinaryOp,bool,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,int8_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_INT8(GrB_Vector,...,GrB_BinaryOp,int8_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,uint8_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_UINT8(GrB_Vector,...,GrB_BinaryOp,uint8_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,int16_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_INT16(GrB_Vector,...,GrB_BinaryOp,int16_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,uint16_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_UINT16(GrB_Vector,...,GrB_BinaryOp,uint16_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,int32_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_INT32(GrB_Vector,...,GrB_BinaryOp,int32_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,uint32_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_UINT32(GrB_Vector,...,GrB_BinaryOp,uint32_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,int64_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_INT64(GrB_Vector,...,GrB_BinaryOp,int64_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,uint64_t,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_UINT64(GrB_Vector,...,GrB_BinaryOp,uint64_t,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,float,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_FP32(GrB_Vector,...,GrB_BinaryOp,float,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,double,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_FP64(GrB_Vector,...,GrB_BinaryOp,double,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,*other*,GrB_Vector,...)	GrB_Vector_apply_BinaryOp1st_UDT(GrB_Vector,...,GrB_BinaryOp,const void*,GrB_Vector,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,GrB_Scalar,...)	GrB_Vector_apply_BinaryOp2nd_Scalar(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,GrB_Scalar,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,bool,...)	GrB_Vector_apply_BinaryOp2nd_BOOL(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,bool,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int8_t,...)	GrB_Vector_apply_BinaryOp2nd_INT8(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int8_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint8_t,...)	GrB_Vector_apply_BinaryOp2nd_UINT8(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint8_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int16_t,...)	GrB_Vector_apply_BinaryOp2nd_INT16(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int16_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint16_t,...)	GrB_Vector_apply_BinaryOp2nd_UINT16(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint16_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int32_t,...)	GrB_Vector_apply_BinaryOp2nd_INT32(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int32_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint32_t,...)	GrB_Vector_apply_BinaryOp2nd_UINT32(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint32_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int64_t,...)	GrB_Vector_apply_BinaryOp2nd_INT64(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,int64_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint64_t,...)	GrB_Vector_apply_BinaryOp2nd_UINT64(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,uint64_t,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,float,...)	GrB_Vector_apply_BinaryOp2nd_FP32(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,float,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,double,...)	GrB_Vector_apply_BinaryOp2nd_FP64(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,double,...)
GrB_apply(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,*other*,...)	GrB_Vector_apply_BinaryOp2nd_UDT(GrB_Vector,...,GrB_BinaryOp,GrB_Vector,const void*,...)

Table 5.8: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_Scalar(GrB_Matrix,...,GrB_BinaryOp,GrB_Scalar,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,bool,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_BOOL(GrB_Matrix,...,GrB_BinaryOp,bool,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,int8_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_INT8(GrB_Matrix,...,GrB_BinaryOp,int8_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,uint8_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_UINT8(GrB_Matrix,...,GrB_BinaryOp,uint8_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,int16_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_INT16(GrB_Matrix,...,GrB_BinaryOp,int16_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,uint16_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_UINT16(GrB_Matrix,...,GrB_BinaryOp,uint16_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,int32_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_INT32(GrB_Matrix,...,GrB_BinaryOp,int32_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,uint32_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_UINT32(GrB_Matrix,...,GrB_BinaryOp,uint32_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,int64_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_INT64(GrB_Matrix,...,GrB_BinaryOp,int64_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,uint64_t,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_UINT64(GrB_Matrix,...,GrB_BinaryOp,uint64_t,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,float,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_FP32(GrB_Matrix,...,GrB_BinaryOp,float,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,double,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_FP64(GrB_Matrix,...,GrB_BinaryOp,double,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,*other*,GrB_Matrix,...)	GrB_Matrix_apply_BinaryOp1st_UDT(GrB_Matrix,...,GrB_BinaryOp,const void*,GrB_Matrix,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,GrB_Scalar,...)	GrB_Matrix_apply_BinaryOp2nd_Scalar(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,GrB_Scalar,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,bool,...)	GrB_Matrix_apply_BinaryOp2nd_BOOL(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,bool,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int8_t,...)	GrB_Matrix_apply_BinaryOp2nd_INT8(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int8_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint8_t,...)	GrB_Matrix_apply_BinaryOp2nd_UINT8(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint8_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int16_t,...)	GrB_Matrix_apply_BinaryOp2nd_INT16(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int16_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint16_t,...)	GrB_Matrix_apply_BinaryOp2nd_UINT16(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint16_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int32_t,...)	GrB_Matrix_apply_BinaryOp2nd_INT32(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int32_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint32_t,...)	GrB_Matrix_apply_BinaryOp2nd_UINT32(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint32_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int64_t,...)	GrB_Matrix_apply_BinaryOp2nd_INT64(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,int64_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint64_t,...)	GrB_Matrix_apply_BinaryOp2nd_UINT64(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,uint64_t,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,float,...)	GrB_Matrix_apply_BinaryOp2nd_FP32(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,float,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,double,...)	GrB_Matrix_apply_BinaryOp2nd_FP64(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,double,...)
GrB_apply(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,*other*,...)	GrB_Matrix_apply_BinaryOp2nd_UDT(GrB_Matrix,...,GrB_BinaryOp,GrB_Matrix,const void*,...)

Table 5.9: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,...)	GrB_Vector_apply_IndexOp_Scalar(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,bool,...)	GrB_Vector_apply_IndexOp_BOOL(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,bool,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int8_t,...)	GrB_Vector_apply_IndexOp_INT8(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int8_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint8_t,...)	GrB_Vector_apply_IndexOp_UINT8(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint8_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int16_t,...)	GrB_Vector_apply_IndexOp_INT16(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int16_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint16_t,...)	GrB_Vector_apply_IndexOp_UINT16(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint16_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int32_t,...)	GrB_Vector_apply_IndexOp_INT32(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int32_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint32_t,...)	GrB_Vector_apply_IndexOp_UINT32(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint32_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int64_t,...)	GrB_Vector_apply_IndexOp_INT64(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int64_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint64_t,...)	GrB_Vector_apply_IndexOp_UINT64(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint64_t,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,float,...)	GrB_Vector_apply_IndexOp_FP32(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,float,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,double,...)	GrB_Vector_apply_IndexOp_FP64(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,double,...)
GrB_apply(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,*other*,...)	GrB_Vector_apply_IndexOp_UDT(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,const void*,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,...)	GrB_Matrix_apply_IndexOp_Scalar(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,bool,...)	GrB_Matrix_apply_IndexOp_BOOL(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,bool,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int8_t,...)	GrB_Matrix_apply_IndexOp_INT8(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int8_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,...)	GrB_Matrix_apply_IndexOp_UINT8(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int16_t,...)	GrB_Matrix_apply_IndexOp_INT16(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int16_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,...)	GrB_Matrix_apply_IndexOp_UINT16(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int32_t,...)	GrB_Matrix_apply_IndexOp_INT32(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int32_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,...)	GrB_Matrix_apply_IndexOp_UINT32(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int64_t,...)	GrB_Matrix_apply_IndexOp_INT64(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int64_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,...)	GrB_Matrix_apply_IndexOp_UINT64(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,float,...)	GrB_Matrix_apply_IndexOp_FP32(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,float,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,double,...)	GrB_Matrix_apply_IndexOp_FP64(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,double,...)
GrB_apply(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,*other*,...)	GrB_Matrix_apply_IndexOp_UDT(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,const void*,...)

Table 5.10: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,...)	GrB_Vector_select_Scalar(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,GrB_Scalar,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,bool,...)	GrB_Vector_select_BOOL(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,bool,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int8_t,...)	GrB_Vector_select_INT8(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int8_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint8_t,...)	GrB_Vector_select_UINT8(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint8_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int16_t,...)	GrB_Vector_select_INT16(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int16_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint16_t,...)	GrB_Vector_select_UINT16(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint16_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int32_t,...)	GrB_Vector_select_INT32(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int32_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint32_t,...)	GrB_Vector_select_UINT32(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint32_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int64_t,...)	GrB_Vector_select_INT64(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,int64_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint64_t,...)	GrB_Vector_select_UINT64(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,uint64_t,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,float,...)	GrB_Vector_select_FP32(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,float,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,double,...)	GrB_Vector_select_FP64(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,double,...)
GrB_select(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,*other*,...)	GrB_Vector_select_UDT(GrB_Vector,...,GrB_IndexUnaryOp,GrB_Vector,const void*,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,...)	GrB_Matrix_select_Scalar(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,GrB_Scalar,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,bool,...)	GrB_Matrix_select_BOOL(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,bool,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int8_t,...)	GrB_Matrix_select_INT8(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int8_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,...)	GrB_Matrix_select_UINT8(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint8_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int16_t,...)	GrB_Matrix_select_INT16(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int16_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,...)	GrB_Matrix_select_UINT16(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint16_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int32_t,...)	GrB_Matrix_select_INT32(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int32_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,...)	GrB_Matrix_select_UINT32(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint32_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int64_t,...)	GrB_Matrix_select_INT64(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,int64_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,...)	GrB_Matrix_select_UINT64(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,uint64_t,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,float,...)	GrB_Matrix_select_FP32(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,float,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,double,...)	GrB_Matrix_select_FP64(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,double,...)
GrB_select(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,*other*,...)	GrB_Matrix_select_UDT(GrB_Matrix,...,GrB_IndexUnaryOp,GrB_Matrix,const void*,...)

Table 5.11: Long-name, nonpolymorphic form of GraphBLAS methods (continued).

Polymorphic signature	Nonpolymorphic signature
GrB_reduce(GrB_Vector,...,GrB_Monoid,...)	GrB_Matrix_reduce_Monoid(GrB_Vector,...,GrB_Monoid,...)
GrB_reduce(GrB_Vector,...,GrB_BinaryOp,...)	GrB_Matrix_reduce_BinaryOp(GrB_Vector,...,GrB_BinaryOp,...)
GrB_reduce(GrB_Scalar,...,GrB_Monoid,GrB_Vector,...)	GrB_Vector_reduce_Monoid_Scalar(GrB_Scalar,...,GrB_Vector,...)
GrB_reduce(GrB_Scalar,...,GrB_BinaryOp,GrB_Vector,...)	GrB_Vector_reduce_BinaryOp_Scalar(GrB_Scalar,...,GrB_Vector,...)
GrB_reduce(bool*,...,GrB_Vector,...)	GrB_Vector_reduce_BOOL(bool*,...,GrB_Vector,...)
GrB_reduce(int8_t*,...,GrB_Vector,...)	GrB_Vector_reduce_INT8(int8_t*,...,GrB_Vector,...)
GrB_reduce(uint8_t*,...,GrB_Vector,...)	GrB_Vector_reduce_UINT8(uint8_t*,...,GrB_Vector,...)
GrB_reduce(int16_t*,...,GrB_Vector,...)	GrB_Vector_reduce_INT16(int16_t*,...,GrB_Vector,...)
GrB_reduce(uint16_t*,...,GrB_Vector,...)	GrB_Vector_reduce_UINT16(uint16_t*,...,GrB_Vector,...)
GrB_reduce(int32_t*,...,GrB_Vector,...)	GrB_Vector_reduce_INT32(int32_t*,...,GrB_Vector,...)
GrB_reduce(uint32_t*,...,GrB_Vector,...)	GrB_Vector_reduce_UINT32(uint32_t*,...,GrB_Vector,...)
GrB_reduce(int64_t*,...,GrB_Vector,...)	GrB_Vector_reduce_INT64(int64_t*,...,GrB_Vector,...)
GrB_reduce(uint64_t*,...,GrB_Vector,...)	GrB_Vector_reduce_UINT64(uint64_t*,...,GrB_Vector,...)
GrB_reduce(float*,...,GrB_Vector,...)	GrB_Vector_reduce_FP32(float*,...,GrB_Vector,...)
GrB_reduce(double*,...,GrB_Vector,...)	GrB_Vector_reduce_FP64(double*,...,GrB_Vector,...)
GrB_reduce(*other*,...,GrB_Vector,...)	GrB_Vector_reduce_UDT(void*,...,GrB_Vector,...)
GrB_reduce(GrB_Scalar,...,GrB_Monoid,GrB_Matrix,...)	GrB_Matrix_reduce_Monoid_Scalar(GrB_Scalar,...,GrB_Monoid,GrB_Matrix,...)
GrB_reduce(GrB_Scalar,...,GrB_BinaryOp,GrB_Matrix,...)	GrB_Matrix_reduce_BinaryOp_Scalar(GrB_Scalar,...,GrB_BinaryOp,GrB_Matrix,...)
GrB_reduce(bool*,...,GrB_Matrix,...)	GrB_Matrix_reduce_BOOL(bool*,...,GrB_Matrix,...)
GrB_reduce(int8_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_INT8(int8_t*,...,GrB_Matrix,...)
GrB_reduce(uint8_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_UINT8(uint8_t*,...,GrB_Matrix,...)
GrB_reduce(int16_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_INT16(int16_t*,...,GrB_Matrix,...)
GrB_reduce(uint16_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_UINT16(uint16_t*,...,GrB_Matrix,...)
GrB_reduce(int32_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_INT32(int32_t*,...,GrB_Matrix,...)
GrB_reduce(uint32_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_UINT32(uint32_t*,...,GrB_Matrix,...)
GrB_reduce(int64_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_INT64(int64_t*,...,GrB_Matrix,...)
GrB_reduce(uint64_t*,...,GrB_Matrix,...)	GrB_Matrix_reduce_UINT64(uint64_t*,...,GrB_Matrix,...)
GrB_reduce(float*,...,GrB_Matrix,...)	GrB_Matrix_reduce_FP32(float*,...,GrB_Matrix,...)
GrB_reduce(double*,...,GrB_Matrix,...)	GrB_Matrix_reduce_FP64(double*,...,GrB_Matrix,...)
GrB_reduce(*other*,...,GrB_Matrix,...)	GrB_Matrix_reduce_UDT(void*,...,GrB_Matrix,...)
GrB_kronecker(GrB_Matrix,...,GrB_Semiring,...)	GrB_Matrix_kronecker_Semiring(GrB_Matrix,...,GrB_Semiring,...)
GrB_kronecker(GrB_Matrix,...,GrB_Monoid,...)	GrB_Matrix_kronecker_Monoid(GrB_Matrix,...,GrB_Monoid,...)
GrB_kronecker(GrB_Matrix,...,GrB_BinaryOp,...)	GrB_Matrix_kronecker_BinaryOp(GrB_Matrix,...,GrB_BinaryOp,...)

Appendix A

Revision history

Changes in 2.0.0 (Released: ##-Xxxxx-2021:

- Reorganized Chapters 2 and 3: Chapter 2 contains prose regarding the basic concepts captured in the API; Chapter 3 presents all of the enumeratiosn, literals, data types, and predefined objects required by the API. Made short captions for the List of Tables.
- (Issue BB-49, BB-50) Updated and corrected language regarding multithreading and completion, and requirements regarding acquire-release memory orders. Methods that used to force complete no longer do.
- (Issue BB-74, BB-9) Assigned integer values to all return codes as well as all enumerations in the API to ensure run-time compatibility between libraries.
- (Issues BB-70, BB-67) Changed semantics and signature of GrB_wait(obj, mode). Added wait modes for 'complete' or 'materialize' and removed GrB_wait(void). This breaks backward compatibility.
- (Issue GH-51) Removed deprecated GrB_SCMP literal from descriptor values. This breaks backward compatibility.
- (Issues BB-8, BB-36) Added sparse GrB_Scalar object and its use in additional variants of extract/setElement methods, and reduce, apply, assign and select operations.
- (Issues BB-34, GH-33, GH-45) Added new select operation that uses an index unary operator. Added new variants of apply that take an index unary operator (matrix and vector variants).
- (Issues BB-68, BB-51) Added serialize and deserialize methods for matrices to/from implementation defined formats.
- (Issues BB-25, GH-42) Added import and export methods for matrices to/from API specified formats. Three formats have been specified: CSC, CSR, COO. Dense row and column formats have been deferred.
- (Issue BB-75) Added matrix constructor to build a diagonal GrB_Matrix from a GrB_Vector.

- (Issue BB-73) Allow GrB_NULL for dup operator in matrix and vector build methods. Return error if duplicate locations encountered.
- (Issue BB-58) Added matrix and vector methods to remove (annihilate) elements.
- (Issue BB-17) Added GrB_ABS_T (absolute value) unary operator.
- (Issue GH-46) Adding GrB_ONEB_T binary operator that returns 1 cast to type T (not to be confused with the proposed unary operator).
- (Issue GH-53) Added language about what constitutes a "conformant" implementation. Added GrB_NOT_IMPLEMENTED return value (API error) for API any combinations of inputs to a method that is not supported by the implementation.
- Added GrB_EMPTY_OBJECT return value (execution error) that is used when an opaque object (currently only GrB_Scalar) is passed as an input that cannot be empty.
- (Issue BB-45) Removed language about annihilators.
- (Issue BB-69) Made names/symbols containing underscores searchable in PDF.
- Updated a number algorithms in the appendix to use new operations and methods.
- Numerous additions (some changes) to the non-polymorphic interface to track changes to the specification.
- Typographical error in version macros was corrected. They are all caps: GRB_VERSION and GRB_SUBVERSION.
- Typographical change to eWiseAdd Description to be consistent in order of set intersections.
- Typographical errors in eWiseAdd: cut-and-paste errors from eWiseMult/set intersection fixed to read eWiseAdd/set union.
- Typographical error (NEQ → NE) in Description of Table 3.8.

Changes in 1.3.0 (Released: 25 September 2019):

- (Issue BB-50) Changed definition of completion and added GrB_wait() that takes an opaque GraphBLAS object as an argument.
- (Issue BB-39) Added GrB_kronecker operation.
- (Issue BB-40) Added variants of the GrB_apply operation that take a binary function and a scalar.
- (Issue BB-59) Changed specification about how reductions to scalar (GrB_reduce) are to be performed (to minimize dependence on monoid identity).
- (Issue BB-24) Added methods to resize matrices and vectors (GrB_Matrix_resize and GrB_Vector_resize).

- (Issue BB-47) Added methods to remove single elements from matrices and vectors (GrB_Matrix_removeEleme and GrB_Vector_removeElement).
- (Issue BB-41) Added GrB_STRUCTURE descriptor flag for masks (consider only the structure of the mask and not the values).
- (Issue BB-64) Deprecated GrB_SCMP in favor of new GrB_COMP for descriptor values.
- (Issue BB-46) Added predefined descriptors covering all possible combinations of field, value pairs.
- Added unary operators: absolute value (GrB_ABS_T) and bitwise complement of integers (GrB_BNOT_I).
- (Issues BB-42, BB-62) Added binary operators: Added boolean exclusive-nor (GrB_LXNOR) and bitwise logical operators on integers (GrB_BOR_I, GrB_BAND_I, GrB_BXOR_I, GrB_BXNOR_I).
- (Issue BB-11) Added a set of predefined monoids and semirings.
- (Issue BB-57) Updated all examples in the appendix to take advantage of new capabilities and predefined objects.
- (Issue BB-43) Added parent-BFS example.
- (Issue BB-1) Fixed bug in the non-batch betweenness centrality algorithm in Appendix C.4 where source nodes were incorrectly assigned path counts.
- (Issue BB-3) Added compile-time preprocessor defines and runtime method for querying the GraphBLAS API version being used.
- (Issue BB-10) Clarified GrB_init() and GrB_finalize() errors.
- (Issue BB-16) Clarified behavior of boolean and integer division. Note that GrB_MINV for integer and boolean types was removed from this version of the spec.
- (Issue BB-19) Clarified aliasing in user-defined operators.
- (Issue BB-20) Clarified language about behavior of GrB_free() with predefined objects (implementation defined)
- (Issue BB-55) Clarified that multiplication does not have to distribute over addition in a GraphBLAS semiring.
- (Issue BB-45) Removed unnecessary language about annihilators.
- (Issue BB-61) Removed unnecessary language about implied zeros.
- (Issue BB-60) Added disclaimer against overspecification.
- Fixed miscellaneous typographical errors (such as $\otimes.\oplus$).

Changes in 1.2.0:

- Removed "provisional" clause.

Changes in 1.1.0:

- Removed unnecessary const from nindices, nrows, and ncols parameters of both extract and assign operations.
- Signature of GrB_UnaryOp_new changed: order of input parameters changed.
- Signature of GrB_BinaryOp_new changed: order of input parameters changed.
- Signature of GrB_Monoid_new changed: removal of domain argument which is now inferred from the domains of the binary operator provided.
- Signature of GrB_Vector_extractTuples and GrB_Matrix_extractTuples to add an in/out argument, n, which indicates the size of the output arrays provided (in terms of number of elements, not number of bytes). Added new execution error, GrB_INSUFFICIENT_SPACE which is returned when the capacities of the output arrays are insufficient to hold all of the tuples.
- Changed GrB_Column_assign to GrB_Col_assign for consistency in non-polymorphic interface.
- Added replace flag (z) notation to Table 4.1.
- Updated the “Mathematical Description" of the assign operation in Table 4.1.
- Added triangle counting example.
- Added subsection headers for accumulate and mask/replace discussions in the Description sections of GraphBLAS operations when the respective text was the “standard" text (i.e., identical in a majority of the operations).
- Fixed typographical errors.

Changes in 1.0.2:

- Expanded the definitions of Vector_build and Matrix_build to conceptually use intermediate matrices and avoid casting issues in certain implementations.
- Fixed the bug in the GrB_assign definition. Elements of the output object are no longer being erased outside the assigned area.
- Changes non-polymorphic interface:
  - Renamed GrB_Row_extract to GrB_Col_extract.
  - Renamed GrB_Vector_reduce_BinaryOp to GrB_Matrix_reduce_BinaryOp.
  - Renamed GrB_Vector_reduce_Monoid to GrB_Matrix_reduce_Monoid.
- Fixed the bugs with respect to isolated vertices in the Maximal Independent Set example.
- Fixed numerous typographical errors.

Appendix B

Non-opaque data format definitions

B.1 GrB_Format: Specify the format for input/output of a GraphBLAS matrix.

In this section, the non-opaque matrix formats specified by GrB_Format and used in matrix import and export methods are defined.

B.1.1 GrB_CSR_FORMAT

The GrB_CSR_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse row (CSR) format. indptr is a pointer to an array of GrB_Index of size nrows+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th row of the matrix. indices is a pointer to an array of number of stored elements (each a GrB_Index), where each element contains the corresponding element's column index within a row of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each row are not required to be sorted by column index.

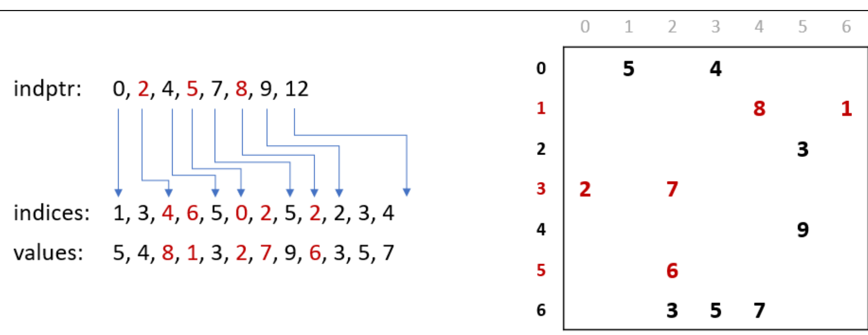

Figure B.1: Data layout for CSR format.

B.1.2 GrB_CSC_FORMAT

The GrB_CSC_FORMAT format indicates that a matrix will be imported or exported using the compressed sparse column (CSC) format. indptr is a pointer to an array of GrB_Index of size ncols+1 elements, where the i'th index will contain the starting index in the values and indices arrays corresponding to the i'th column of the matrix. indices is a pointer to an array of number of stored elements (each a GrB_Index), where each element contains the corresponding element's row index within a column of the matrix. values is a pointer to an array of number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. The elements of each column are not required to be sorted by row index.

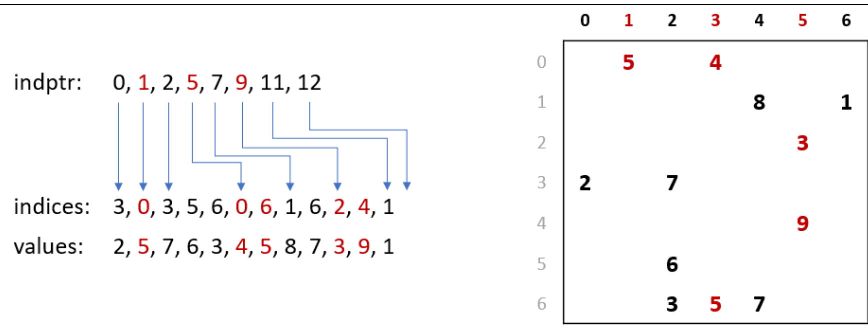

Figure B.2: Data layout for CSC format.

B.1.3 GrB_COO_FORMAT

The GrB_COO_FORMAT format indicates that a matrix will be imported or exported using the coordinate list (COO) format. indptr is a pointer to an array of GrB_Index of size number of stored elements, where each element contains the corresponding element's column index. indices will be a pointer to an array of GrB_Index of size number of stored elements, where each element contains the corresponding element's row index. values will be a pointer to an array of size number of stored elements (each the size of the scalar stored in the matrix) containing the corresponding value. Elements are not required to be sorted in any order.

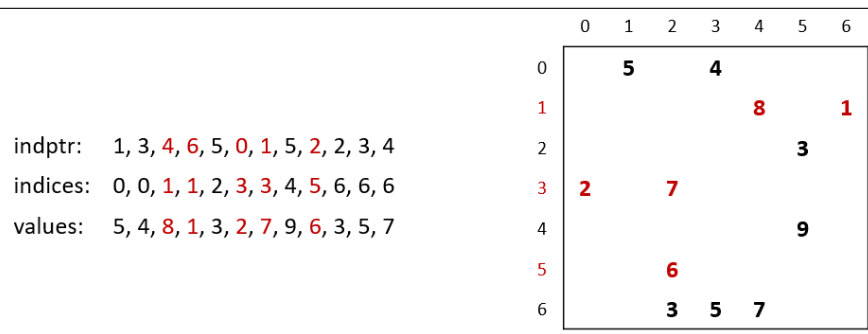

Figure B.3: Data layout for COO format.

Appendix C

Examples

C.1 Example: Level breadth-first search (BFS) in GraphBLAS

```c
#include <stdlib.h>
#include <stdio.h>
#include <stdint.h>
#include <stdbool.h>
#include "GraphBLAS.h"

/*
 * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
 * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
 * If i is not reacheable from s, then v[i] = 0. (Vector v should be empty on input.)
 */
GrB_Info BFS(GrB_Vector *v, GrB_Matrix A, GrB_Index s)
{
  GrB_Index n;
  GrB_Matrix_nrows(&n,A);                          // n = # of rows of A

  GrB_Vector_new(v,GrB_INT32,n);                   // Vector<int32_t> v(n)

  GrB_Vector q;                                    // vertices visited in each level
  GrB_Vector_new(&q,GrB_BOOL,n);                   // Vector<bool> q(n)
  GrB_Vector_setElement(q,(bool)true,s);           // q[s] = true, false everywhere else

  /*
   * BFS traversal and label the vertices.
   */
  int32_t d = 0;                                   // d = level in BFS traversal
  bool succ = false;                               // succ == true when some successor found
  do {
    ++d;                                           // next level (start with 1)
    GrB_assign(*v,q,GrB_NULL,d,GrB_ALL,n,GrB_NULL);   // v[q] = d
    GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
            q,A,GrB_DESC_RC);                      // q[!v] = q ||.&& A ; finds all the
                                                   // unvisited successors from current q
    GrB_reduce(&succ,GrB_NULL,GrB_LOR_MONOID_BOOL,
               q,GrB_NULL);                        // succ = ||(q)
  } while (succ);                                  // if there is no successor in q, we are done.

  GrB_free(&q);                                    // q vector no longer needed

  return GrB_SUCCESS;
}
```

C.2 Example: Level BFS in GraphBLAS using apply

```c
#include <stdlib.h>
#include <stdio.h>
#include <stdint.h>
#include <stdbool.h>
#include "GraphBLAS.h"

/*
 * Given a boolean n x n adjacency matrix A and a source vertex s, performs a BFS traversal
 * of the graph and sets v[i] to the level in which vertex i is visited (v[s] == 1).
 * If i is not reachable from s, then v[i] does not have a stored element.
 * Vector v should be uninitialized on input.
 */
GrB_Info BFS(GrB_Vector *v, const GrB_Matrix A, GrB_Index s)
{
  GrB_Index n;
  GrB_Matrix_nrows(&n,A);                          // n = # of rows of A

  GrB_Vector_new(v,GrB_INT32,n);                   // Vector<int32_t> v(n) = 0

  GrB_Vector q;                                    // vertices visited in each level
  GrB_Vector_new(&q,GrB_BOOL,n);                   // Vector<bool> q(n) = false
  GrB_Vector_setElement(q,(bool)true,s);           // q[s] = true, false everywhere else

  /*
   * BFS traversal and label the vertices.
   */
  int32_t level = 0;                               // level = depth in BFS traversal
  GrB_Index nvals;
  do {
    ++level;                                       // next level (start with 1)
    GrB_apply(*v,GrB_NULL,GrB_PLUS_INT32,
              GrB_SECOND_INT32,q,level,GrB_NULL);  // v[q] = level
    GrB_vxm(q,*v,GrB_NULL,GrB_LOR_LAND_SEMIRING_BOOL,
            q,A,GrB_DESC_RC);                      // q[!v] = q ||.&& A ; finds all the
                                                   // unvisited successors from current q
    GrB_Vector_nvals(&nvals, q);
  } while (nvals);                                 // if there is no successor in q, we are done.

  GrB_free(&q);                                    // q vector no longer needed

  return GrB_SUCCESS;
}
```

C.3 Example: Parent BFS in GraphBLAS

```c
#include <stdlib.h>
#include <stdio.h>
#include <stdint.h>
#include <stdbool.h>
#include "GraphBLAS.h"

/*
 * Given a binary n x n adjacency matrix A and a source vertex s, performs a BFS
 * traversal of the graph and sets parents[i] to the index of vertex i's parent.
 * The parent of the root vertex, s, will be set to itself (parents[s] == s). If
 * vertex i is not reachable from s, parents[i] will not contain a stored value.
 */
GrB_Info BFS(GrB_Vector *parents, const GrB_Matrix A, GrB_Index s)
{
  GrB_Index N;
  GrB_Matrix_nrows(&N, A);                        // N = # vertices

  GrB_Vector_new(parents, GrB_UINT64, N);
  GrB_Vector_setElement(*parents, s, s);          // parents[s] = s

  GrB_Vector wavefront;
  GrB_Vector_new(&wavefront, GrB_UINT64, N);
  GrB_Vector_setElement(wavefront, 1UL, s);       // wavefront[s] = 1

  /*
   * BFS traversal and label the vertices.
   */
  GrB_Index nvals;
  GrB_Vector_nvals(&nvals, wavefront);

  while (nvals > 0)
  {
    // convert all stored values in wavefront to their 0-based index
    GrB_apply(wavefront, GrB_NULL, GrB_NULL, GrB_ROWINDEX_INT64,
              wavefront, 0UL, GrB_NULL);

    // "FIRST" because left-multiplying wavefront rows. Masking out the parent
    // list ensures wavefront values do not overwrite parents already stored.
    GrB_vxm(wavefront, *parents, GrB_NULL, GrB_MIN_FIRST_SEMIRING_UINT64,
            wavefront, A, GrB_DESC_RSC);

    // Don't need to mask here since we did it in mxm. Merges new parents in
    // current wavefront with existing parents: parents += wavefront
    GrB_apply(*parents, GrB_NULL, GrB_PLUS_UINT64,
              GrB_IDENTITY_UINT64, wavefront, GrB_NULL);

    GrB_Vector_nvals(&nvals, wavefront);
  }

  GrB_free(&wavefront);

  return GrB_SUCCESS;
}
```

C.4 Example: Betweenness centrality (BC) in GraphBLAS

```c
#include <stdlib.h>
#include <stdio.h>
#include <stdint.h>
#include <stdbool.h>
#include "GraphBLAS.h"

/*
 * Given a boolean n x n adjacency matrix A and a source vertex s,
 * compute the BC-metric vector delta, which should be empty on input.
 */
GrB_Info BC(GrB_Vector *delta, GrB_Matrix A, GrB_Index s)
{
  GrB_Index n;
  GrB_Matrix_nrows(&n,A);                        // n = # of vertices in graph

  GrB_Vector_new(delta,GrB_FP32,n);              // Vector<float> delta(n)

  GrB_Matrix sigma;                              // Matrix<int32_t> sigma(n,n)
  GrB_Matrix_new(&sigma,GrB_INT32,n,n);          // sigma[d,k] = #shortest paths to node k at level d

  GrB_Vector q;
  GrB_Vector_new(&q, GrB_INT32, n);              // Vector<int32_t> q(n) of path counts
  GrB_Vector_setElement(q,1,s);                  // q[s] = 1

  GrB_Vector p;                                  // Vector<int32_t> p(n) shortest path counts so far
  GrB_Vector_dup(&p, q);                         // p = q

  GrB_vxm(q,p,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_INT32,
          q,A,GrB_DESC_RC);                      // get the first set of out neighbors

  /*
   * BFS phase
   */
  GrB_Index d = 0;                               // BFS level number
  int32_t sum = 0;                               // sum == 0 when BFS phase is complete

  do {
    GrB_assign(sigma,GrB_NULL,GrB_NULL,q,d,GrB_ALL,n,GrB_NULL);     // sigma[d,:] = q
    GrB_eWiseAdd(p,GrB_NULL,GrB_NULL,GrB_PLUS_INT32,p,q,GrB_NULL); // accum path counts on this level
    GrB_vxm(q,p,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_INT32,
            q,A,GrB_DESC_RC);                                       // q = # paths to nodes reachable
                                                                    //     from current level
    GrB_reduce(&sum,GrB_NULL,GrB_PLUS_MONOID_INT32,q,GrB_NULL);     // sum path counts at this level
    ++d;
  } while (sum);

  /*
   * BC computation phase
   * (t1,t2,t3,t4) are temporary vectors
   */
  GrB_Vector t1; GrB_Vector_new(&t1,GrB_FP32,n);
  GrB_Vector t2; GrB_Vector_new(&t2,GrB_FP32,n);
  GrB_Vector t3; GrB_Vector_new(&t3,GrB_FP32,n);
  GrB_Vector t4; GrB_Vector_new(&t4,GrB_FP32,n);

  for(int i=d-1; i>0; i--)
  {
    GrB_assign(t1,GrB_NULL,GrB_NULL,1.0f,GrB_ALL,n,GrB_NULL);           // t1 = 1+delta
    GrB_eWiseAdd(t1,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,t1,*delta,GrB_NULL);
    GrB_extract(t2,GrB_NULL,GrB_NULL,sigma,GrB_ALL,n,i,GrB_DESC_T0);    // t2 = sigma[i,:]
    GrB_eWiseMult(t2,GrB_NULL,GrB_NULL,GrB_DIV_FP32,t1,t2,GrB_NULL);    // t2 = (1+delta)/sigma[i,:]
    GrB_mxv(t3,GrB_NULL,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_FP32,          // add contributions made by
```

```
                 A, t2, GrB_NULL);                                      //      successors of a node
      GrB_extract(t4, GrB_NULL, GrB_NULL, sigma, GrB_ALL, n, i-1, GrB_DESC_T0);  // t4 = sigma[i-1,:]
      GrB_eWiseMult(t4, GrB_NULL, GrB_NULL, GrB_TIMES_FP32, t4, t3, GrB_NULL);  // t4 = sigma[i-1,:]*t3
      GrB_eWiseAdd(*delta, GrB_NULL, GrB_NULL, GrB_PLUS_FP32, *delta, t4, GrB_NULL); // accumulate into delta
    }

    GrB_free(&sigma);
    GrB_free(&q);  GrB_free(&p);
    GrB_free(&t1);  GrB_free(&t2);  GrB_free(&t3);  GrB_free(&t4);

    return GrB_SUCCESS;
}
```

C.5 Example: Batched BC in GraphBLAS

```c
#include <stdlib.h>
#include "GraphBLAS.h"  // in addition to other required C headers

// Compute partial BC metric for a subset of source vertices, s, in graph A
GrB_Info BC_update(GrB_Vector *delta, GrB_Matrix A, GrB_Index *s, GrB_Index nsver)
{
  GrB_Index n;
  GrB_Matrix_nrows(&n, A);                              // n = # of vertices in graph
  GrB_Vector_new(delta,GrB_FP32,n);                     // Vector<float> delta(n)

  // index and value arrays needed to build numsp
  GrB_Index *i_nsver = (GrB_Index*)malloc(sizeof(GrB_Index)*nsver);
  int32_t   *ones    = (int32_t*)  malloc(sizeof(int32_t)*nsver);
  for(int i=0; i<nsver; ++i) {
    i_nsver[i] = i;
    ones[i] = 1;
  }

  // numsp: structure holds the number of shortest paths for each node and starting vertex
  // discovered so far.  Initialized to source vertices:  numsp[s[i],i]=1, i=[0,nsver)
  GrB_Matrix numsp;
  GrB_Matrix_new( ,GrB_INT32,n,nsver);
  GrB_Matrix_build(numsp,s,i_nsver,ones,nsver,GrB_PLUS_INT32);
  free(i_nsver); free(ones);

  // frontier: Holds the current frontier where values are path counts.
  // Initialized to out vertices of each source node in s.
  GrB_Matrix frontier;
  GrB_Matrix_new(&frontier,GrB_INT32,n,nsver);
  GrB_extract(frontier,numsp,GrB_NULL,A,GrB_ALL,n,s,nsver,GrB_DESC_RCT0);

  // sigma: stores frontier information for each level of BFS phase.  The memory
  // for an entry in sigmas is only allocated within the do-while loop if needed.
  // n is an upper bound on diameter.
  GrB_Matrix *sigmas = (GrB_Matrix*)malloc(sizeof(GrB_Matrix)*n);

  int32_t   d = 0;                                      // BFS level number
  GrB_Index nvals = 0;                                  // nvals == 0 when BFS phase is complete

  // -------------------- The BFS phase (forward sweep) --------------------------
  do {
    // sigmas[d](:,s) = d^th level frontier from source vertex s
    GrB_Matrix_new(&(sigmas[d]),GrB_BOOL,n,nsver);

    GrB_apply(sigmas[d],GrB_NULL,GrB_NULL,
              GrB_IDENTITY_BOOL,frontier,GrB_NULL);     // sigmas[d](:,:) = (Boolean) frontier
    GrB_eWiseAdd(numsp,GrB_NULL,GrB_NULL,GrB_PLUS_INT32,
                 numsp,frontier,GrB_NULL);              // numsp += frontier (accum path counts)
    GrB_mxm(frontier,numsp,GrB_NULL,GrB_PLUS_TIMES_SEMIRING_INT32,
            A,frontier,GrB_DESC_RCT0);                  // f<!numsp> = A' +.* f (update frontier)
    GrB_Matrix_nvals(&nvals,frontier);                  // number of nodes in frontier at this level
    d++;
  } while (nvals);

  // nspinv: the inverse of the number of shortest paths for each node and starting vertex.
  GrB_Matrix nspinv;
  GrB_Matrix_new(&nspinv,GrB_FP32,n,nsver);
  GrB_apply(nspinv,GrB_NULL,GrB_NULL,
            GrB_MINV_FP32,numsp,GrB_NULL);              // nspinv = 1./numsp

  // bcu: BC updates for each vertex for each starting vertex in s
  GrB_Matrix bcu;
```

```
  GrB_Matrix_new(&bcu,GrB_FP32,n,nsver);
  GrB_assign(bcu,GrB_NULL,GrB_NULL,
             1.0f,GrB_ALL,n,GrB_ALL,nsver,GrB_NULL);   // filled with 1 to avoid sparsity issues

  GrB_Matrix w;                                        // temporary workspace matrix
  GrB_Matrix_new(&w,GrB_FP32,n,nsver);

  // -------------------- Tally phase (backward sweep) --------------------
  for (int i=d-1; i>0; i--)  {
    GrB_eWiseMult(w,sigmas[i],GrB_NULL,
                  GrB_TIMES_FP32,bcu,nspinv,GrB_DESC_R);   // w<sigmas[i]>=(1 ./ nsp).*bcu

    // add contributions by successors and mask with that BFS level's frontier
    GrB_mxm(w,sigmas[i-1],GrB_NULL,GrB_PLUS_TIMES_SEMIRING_FP32,
            A,w,GrB_DESC_R);                               // w<sigmas[i-1]> = (A +.* w)
    GrB_eWiseMult(bcu,GrB_NULL,GrB_PLUS_FP32,GrB_TIMES_FP32,
                  w,numsp,GrB_NULL);                       // bcu += w .* numsp
  }

  // row reduce bcu and subtract "nsver" from every entry to account
  // for 1 extra value per bcu row element.
  GrB_reduce(*delta,GrB_NULL,GrB_NULL,GrB_PLUS_FP32,bcu,GrB_NULL);
  GrB_apply(*delta,GrB_NULL,GrB_NULL,GrB_MINUS_FP32,*delta,(float)nsver,GrB_NULL);

  // Release resources
  for(int i=0; i<d; i++) {
    GrB_free(&(sigmas[i]));
  }
  free(sigmas);

  GrB_free(&frontier);     GrB_free( );
  GrB_free(&nspinv);       GrB_free(&bcu);       GrB_free(&w);

  return GrB_SUCCESS;
}
```

C.6 Example: Maximal independent set (MIS) in GraphBLAS

```c
#include <stdlib.h>
#include <stdio.h>
#include <stdint.h>
#include <stdbool.h>
#include "GraphBLAS.h"

// Assign a random number to each element scaled by the inverse of the node's degree.
// This will increase the probability that low degree nodes are selected and larger
// sets are selected.
void setRandom(void *out, const void *in)
{
  uint32_t degree = *(uint32_t*)in;
  *(float*)out = (0.0001f + random()/(1. + 2.*degree)); // add 1 to prevent divide by zero
}

/*
 * A variant of Luby's randomized algorithm [Luby 1985].
 *
 * Given a numeric n x n adjacency matrix A of an unweighted and undirected graph (where
 * the value true represents an edge), compute a maximal set of independent vertices and
 * return it in a boolean n-vector, 'iset' where set[i] == true implies vertex i is a member
 * of the set (the iset vector should be uninitialized on input.)
 */
GrB_Info MIS(GrB_Vector *iset, const GrB_Matrix A)
{
  GrB_Index n;
  GrB_Matrix_nrows(&n,A);                         // n = # of rows of A

  GrB_Vector prob;                                // holds random probabilities for each node
  GrB_Vector neighbor_max;                        // holds value of max neighbor probability
  GrB_Vector new_members;                         // holds set of new members to iset
  GrB_Vector new_neighbors;                       // holds set of new neighbors to new iset mbrs.
  GrB_Vector candidates;                          // candidate members to iset

  GrB_Vector_new(&prob,GrB_FP32,n);
  GrB_Vector_new(&neighbor_max,GrB_FP32,n);
  GrB_Vector_new(&new_members,GrB_BOOL,n);
  GrB_Vector_new(&new_neighbors,GrB_BOOL,n);
  GrB_Vector_new(&candidates,GrB_BOOL,n);
  GrB_Vector_new(iset,GrB_BOOL,n);                // Initialize independent set vector, bool

  GrB_UnaryOp set_random;
  GrB_UnaryOp_new(&set_random,setRandom,GrB_FP32,GrB_UINT32);

  // compute the degree of each vertex.
  GrB_Vector degrees;
  GrB_Vector_new(&degrees,GrB_FP64,n);
  GrB_reduce(degrees,GrB_NULL,GrB_NULL,GrB_PLUS_FP64,A,GrB_NULL);

  // Isolated vertices are not candidates: candidates[degrees != 0] = true
  GrB_assign(candidates,degrees,GrB_NULL,true,GrB_ALL,n,GrB_NULL);

  // add all singletons to iset: iset[degree == 0] = 1
  GrB_assign(*iset,degrees,GrB_NULL,true,GrB_ALL,n,GrB_DESC_RC) ;

  // Iterate while there are candidates to check.
  GrB_Index nvals;
  GrB_Vector_nvals(&nvals, candidates);
  while (nvals > 0) {
    // compute a random probability scaled by inverse of degree
    GrB_apply(prob,candidates,GrB_NULL,set_random,degrees,GrB_DESC_R);

```

```
    // compute the max probability of all neighbors
    GrB_mxv(neighbor_max, candidates, GrB_NULL, GrB_MAX_SECOND_SEMIRING_FP32, A, prob, GrB_DESC_R);

    // select vertex if its probability is larger than all its active neighbors,
    // and apply a "masked no-op" to remove stored falses
    GrB_eWiseAdd(new_members, GrB_NULL, GrB_NULL, GrB_GT_FP64, prob, neighbor_max, GrB_NULL);
    GrB_apply(new_members, new_members, GrB_NULL, GrB_IDENTITY_BOOL, new_members, GrB_DESC_R);

    // add new members to independent set.
    GrB_eWiseAdd(*iset, GrB_NULL, GrB_NULL, GrB_LOR, *iset, new_members, GrB_NULL);

    // remove new members from set of candidates c = c & !new
    GrB_eWiseMult(candidates, new_members, GrB_NULL,
                  GrB_LAND, candidates, candidates, GrB_DESC_RC);

    GrB_Vector_nvals(&nvals, candidates);
    if (nvals == 0) { break; }                // early exit condition

    // Neighbors of new members can also be removed from candidates
    GrB_mxv(new_neighbors, candidates, GrB_NULL, GrB_LOR_LAND_SEMIRING_BOOL,
            A, new_members, GrB_NULL);
    GrB_eWiseMult(candidates, new_neighbors, GrB_NULL, GrB_LAND,
                  candidates, candidates, GrB_DESC_RC);

    GrB_Vector_nvals(&nvals, candidates);
  }

  GrB_free(&neighbor_max);                    // free all objects "new'ed"
  GrB_free(&new_members);
  GrB_free(&new_neighbors);
  GrB_free(&prob);
  GrB_free(&candidates);
  GrB_free(&set_random);
  GrB_free(&degrees);

  return GrB_SUCCESS;
}
```

C.7 Example: Counting triangles in GraphBLAS

```
#include <stdlib.h>
#include <stdio.h>
#include <stdint.h>
#include <stdbool.h>
#include "GraphBLAS.h"

/*
 * Given an n x n boolean adjacency matrix, A, of an undirected graph, computes
 * the number of triangles in the graph.
 */
uint64_t triangle_count(GrB_Matrix A)
{
  GrB_Index n;
  GrB_Matrix_nrows(&n, A);                         // n = # of vertices

  // L: NxN, lower-triangular, bool
  GrB_Matrix L;
  GrB_Matrix_new(&L, GrB_BOOL, n, n);
  GrB_select(L, GrB_NULL, GrB_NULL, GrB_TRIL, A, 0UL, GrB_NULL);

  GrB_Matrix C;
  GrB_Matrix_new(&C, GrB_UINT64, n, n);

  GrB_mxm(C, L, GrB_NULL, GrB_PLUS_TIMES_SEMIRING_UINT64, L, L, GrB_NULL); // C<L> = L +.* L

  uint64_t count;
  GrB_reduce(&count, GrB_NULL, GrB_PLUS_MONOID_UINT64, C, GrB_NULL);      // 1-norm of C

  GrB_free(&C);
  GrB_free(&L);

  return count;
}
```